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Cooperative Adaptive Cruise Control using Pontraygin's Minimum Principle: An Optimal Control

by

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Dedicated to:

Parents:

Abdul Latif

Khalida Latif

Brothers:

Sohail Latif (Late)

Sohaib Latif (Late)

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Abstract

In the last century, growth in technology, urbanization and rapid increase of population raised many problems. Trend of personal vehicle has also been increased. There are number of auto-mobile companies around the globe which are increasing the load of vehicles exponentially. Due to these reasons, congestion and traffic jam have been increased especially on highways. In addition, these congestions have harmful effects on environment. The increase in highway intensity leads to road accidents and irregularity in traffic which increase fuel consumption rate.

A lot of research has been done to design an intelligent transport system to address the above problems. The intelligent transport is a system in which vehicles sense their surrounding, take decisions independently and perform control action or help the driver in controlling vehicle. The motivation for designing an intelligent transport system is to increase the convenience of driver in driving task.

On highways, to increase the road capacity, vehicles need to be driven close to each other on a safe distance. This thesis focuses on cooperative adaptive cruise control (CACC), which takes data from environment and maintains safe distance from the preceding vehicle. This safe distance may be decided by the designer or can be kept adaptive. This system measure distance from the preceding vehicle through radar and some other sensors and also take information such as position, velocity and acceleration of preceding vehicle through wireless communication. This information helps it to decide the next control action. In the form of platoon, all vehicles are communicating with leader and other vehicles in front of them and maintain smaller headway.

Proportional integral derivative (PID) controller has been widely used in research as well as in practical scenario with some refinement for comfort and gear shift. Model predictive control (MPC) and Protraygin's minimum principle (PMP) has also been used in literature.

In literature, a simple vehicle model has been used for CACC. Engine's model has not been considered along the kinematic model of vehicle. In this work, engine's states along the kinematic model of vehicle has been used. The Pontragyin's minimum principle is used to minimize the objective/cost function of vehicle following policy. A numerical algorithm has been proposed to solve the optimization problem. An important aspect of this control is that, the computational complexities do not increase exponentially with the increase in complexity of system model.

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Abbreviations

CC	Cruise control
ACC	Adaptive cruise control
CACC	Cooperative adaptive cruise control
PATH	Partners for advanced transportation technology
V2V	Vehicle to vehicle
V2I	Vehicle to infrastructure
GPS	Global positioning system
MPC	Model predictive control
PID	Proportional integral control
PMP	Pontraygin's minimum principle
DSRC	Dedicated short range communications
DKF	Discrete Kalman filter
LQE	Linear quadratic estimator
NHTSA	National highway traffic safety administration
SEA	Society of automotive engineering
MVEM	Mean value engine model
AFR	Air fuel ratio

Chapter 1

Introduction

Autonomous or self-driving vehicles provide an opportunity for driver to chose between self-drive and manual mode. This precedent feature has an huge impact on automotive industry, and provides an opportunity to perform most challenging task to perform through controlled system. The feasibility of secure self-driving vehicle is still questionable among experts.

1.1 Historic Change in Automotive Industry

Automotive industry has advanced in very evolutionary manner. Now, the vehicles have become modern, safe and secure, but all this happens in stepwise fashion. This leads the automotive industry on the brink of the new era which would change the basic definition of travelling, in which no driver is needed to drive the car [1]. The competition has been raised between conventional and non-automotive car makers to do more research, design prototypes and present something new to lead the market [2].

1.2 Why Autonomous Driving?

19th century saw a great growth in population, hence the urbanization has increased. The movement from rural to urban area effected millions of people around the world by creating mega cities. This result in severe traffic congestion, increased waiting time in rush hours, air and noise pollution and problem in finding parking spots, urge the need of an alternative smart solution for personal mobility [3, 4]. If autonomous car resolve all these issues, it can surely grab one-third of the world's market [5].

1.3 Autonomous Vehicle

An autonomous car also called driver less vehicle is such type of car, which is capable of sensing its surrounding environment, using special type of sensors and navigates around by itself or without human input as shown in figure 1.1.

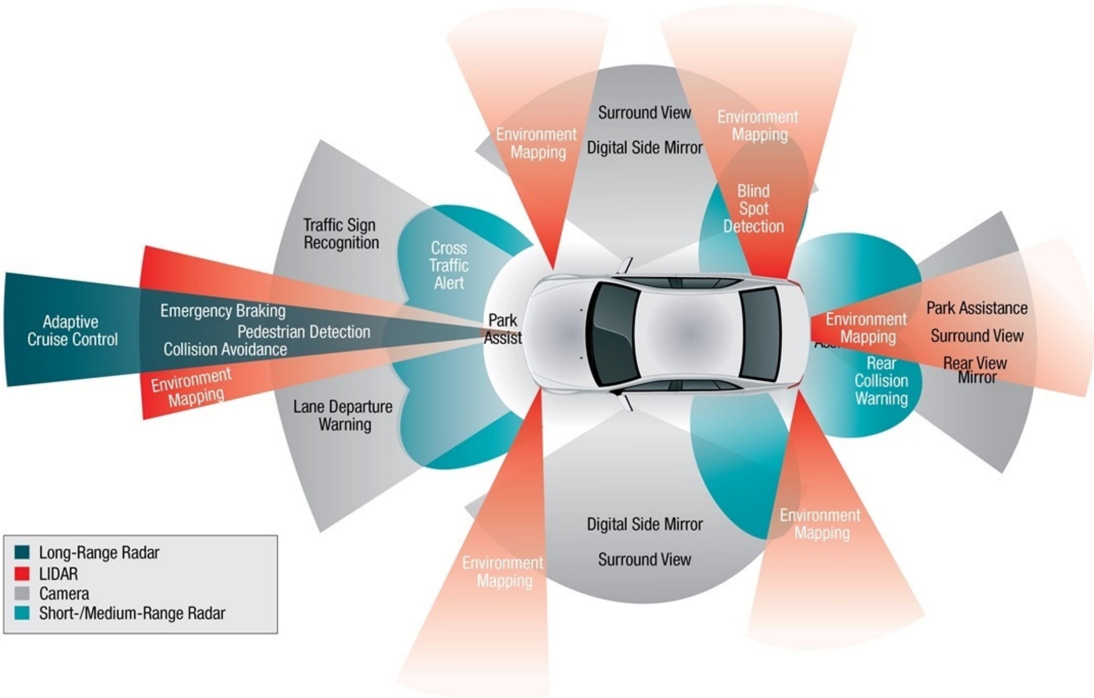

Figure 1.1: Autonomous Vehicle [6]

For sensing the surroundings, different techniques can be used to map near areas, such as radar, global positioning sysyem (GPS), odometer, laser light and computer vision algorithms. The data received from these sensors translated to navigational paths by using advanced algorithms [4].

The concept of autonomous car is associated with many potential benefits like reduced mobility, enhanced safety features which reduce crime and increase the customer satisfaction [7]. It also reduces the accident rate drastically in urban environment, results in reduced medical and insurance cost. Autonomous cars are expected to increase the traffic flow, facilitate the elders, disables and children in movement [8]. It will also solve the problems of congestion and parking spaces. In short, it will change the whole business model of transportation [7].

1.4 History of Autonomous Vehicles

The experimentations on autonomous vehicles dated back to 1920s when first attempt was being made for self-driving vehicle, but fruitful test took place around 1950s [9]. The first realistic model of self-driving vehicle was appeared in 1980s with the coordination of Carnegie Mellon Universitys Navlab and autonomous land vehicle (ALV) project [10]. After wards, many other companies and research institutes have developed their own prototypes. In 2015, some new companies in US states of Nevada, Virginia, California and Michigan with the cooperation of Washington allowed for testing of self-driving vehicles on road [11].

In 2017, Audi introduced its newest model of A8 series, which would be autonomous under the speed of 60 km/h. It gives provision to the driver not to check safety regularly. It is claimed that Audi A8 would be the first vehicle to have level 3 autonomous driving and it will be first ever car of its kind to use laser scanner with integrated cameras and ultrasonic sensors for navigation [11].

1.5 Autonomous vs Automated

Autonomous means self-sufficient. Autonomous control means to work under uncertain environment without use of external assistance. One method is to develop a communication channel between the vehicles present in the vicinity, to avoid collision and to reduce congestion. Such methods are effective since they do not require any human input [12]. This thesis will segregate between the word autonomous and automated. The term autonomous is more meaningful because it refers to complete autonomy of the vehicle, it can make self-decision when required. Whereas, the automated control is described by any operation done by the machine. Most of today's vehicles are automated, as they need a driver to observe the surrounding and they do not have the ability to reach on destination independently. Recent developments are being made to make vehicles more and more autonomous [13].

1.6 Classification of Autonomous Drive

There are six different levels of automation, which range from no automation to fully automated (autonomous) vehicle. This concept was published in society of automotive engineers (SAE) international 2014, an automotive standardization as J3016 [14]. This classification based on level of involvement of human driver, rather than involving vehicle capabilities.

1.6.1 Levels of Driving Automation

While talking about the automatic driving, the first step to determine the type of automatic system. There are two main bodies, National Highway Traffic Safety Administration (NHTSA) of the United States and SAE, who define the classification of automation [15]. The main difference is that NHTSA used a five-point scale to define the automatic driving step, while the SAE used six steps.

The following SAE standard has also been accepted by NHTSA.

TABLE 1.1: Levels of autonomous drive [16]

Level	**Degree of automation**	**Driver's responsibilities**	**Timeline**
0	No automation	Complete driving task	Already exist
1	Driver assistance	Except one task e.g. acceleration and braking, rest all the responsibilities are of driver	Already exist
2	Partial automation	Driver can take relief from throttle, braking and steering while moving on highways only	Already exist
3	Conditional automation	Driver can take relief from throttle, brake and steering in urban environment as well	Already exist but commercially unavailable
4	High automation	Driver is required only for case of emergency	Exist, but not allowed on public roads
5	Full self driving	Just need to set destination and some other specifications	Next 2 years

Figure 1.2 is showing the levels of automation from human control till machine controlled.

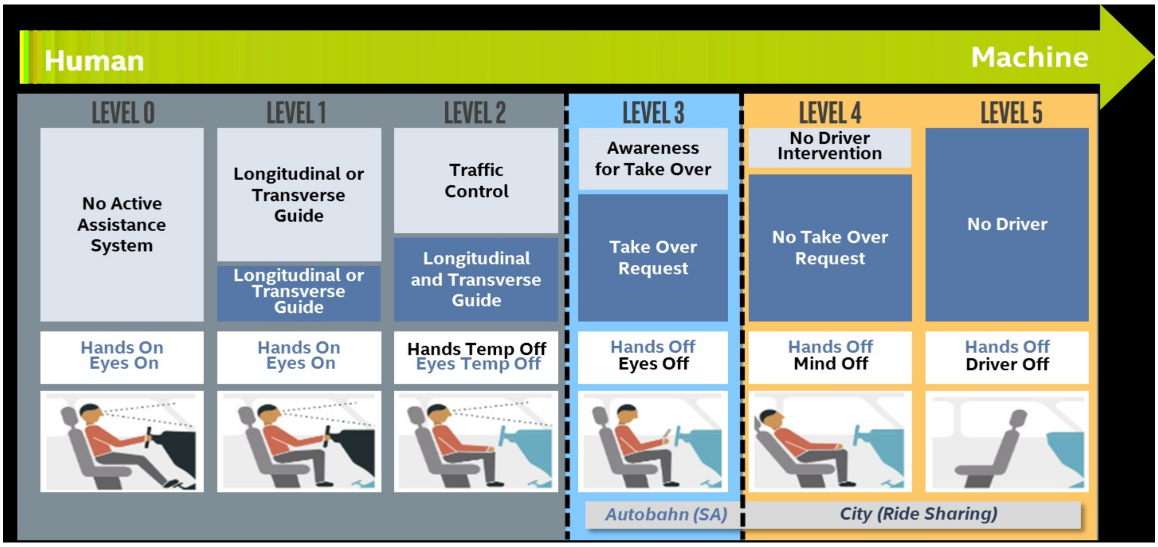

FIGURE 1.2: Levels of automation by SEA [16]

In autonomous vehicle, there are multiple control systems e.g. anti collision system, overtaking warning, lane warning, blind sport detection, speed control system etc are working to control the vehicle. The speed control of an autonomous vehicle is one of the most difficult automation challenges because of constraints on mobility, speed of motion, high-speed operation, complex interaction with the environment and typically a lack of prior information. The vehicle control can be separated into lateral and longitudinal controls. Here we focus on the longitudinal speed control of the vehicle to follow the leader with a minimum of track error.

1.7 Speed Control System

In 1788, James Watt and Matthew Boulton controlled the speed of steam engine. By using different loads, they adjusted the throttle and maintain almost a constant speed [17]. Speed control is used by Wilson-Pilcher in 1900 and then by Peerless in 1910s, in automobiles. Peerless claimed that while running on downhill or moving uphill, his system is able to maintain almost a constant speed. With the passage of time, new technologies opened new ways to control the speed according to requirements and benefits [18]. Some important inventions and techniques to control the speed of vehicles are discussed below.

1.7.1 Cruise Control

Cruise control (CC) or auto-cruise is based on servomechnaic system, which takes the control of throttle and moves the vehicle on a specific speed set by the driver [19]. Driver needs to bring the vehicle on the specified speed and then give control to CC system to drive the vehicle. CC system takes reading from the engine speedometer, sensors mounted on wheel and engine rpm, and generates an electronic signal which is used to drive a motor connected with throttle [20]. Most of the systems do not allow the cruise below a specific speed which is typically 40 km/h (25 mph).

All cruise systems are able to tune both explicitly and automatically, when driver applies brake and slows down the vehicle. It has memory backup to retrieve the set speed after braking and also allows to change the set speed without applying brake. On CC mode, the driver can take control of pedal to accelerate and increase the speed but vehicle will reduce speed to set point after leaving the pedal [21].

In modern vehicles having electronic throttle system, cruise control can easily be integrated in engine management system. Modern cruise control system (adaptive cruise control) has ability to change set point depending on the distance from the vehicle in front of it [22].

1.7.1.1 Advantages of CC

It is useful on highways as it reduces fatigue and enable drive to change its poster safely. It restricts from over speeding to those drivers who increase speed unconsciously. Its speed limiter function do not let the vehicle to accelerate more than a specific acceleration, which is useful while driving downhill.

1.7.1.2 Disadvantages of CC

Inappropriate use of cruise control leads to accident because speed do not decrease on curves. It does not reduce speed on rough and loose terrain. It reduces the traction in wet and rainy weather. It do not stop or reduce speed of vehicle if preceder stop or reduces its speed.

1.7.2 Adaptive Cruise Control

Adaptive cruise control (ACC) is a system which adjusts the speed of vehicle automatically and maintains a safe distance from preceding vehicle. On board sensors like radar, laser and stereo cameras are used to maintain a specific distance from the leading vehicle [22]. In intelligent cars of future generation, ACC technology is a key component. It increases the road capacity, road safety and reduces driver error by maintaining an optimum distance between vehicles. According to SAE international, vehicle having adaptive cruise control is considered a Level-1 autonomous vehicle [14].

In ACC, just like cruise control, driver sets the maximum speed limit and ACC system instructs the car to stay on a specific distance [23]. Separation between the vehicles can be distance base (fix distance) or time base (distance between vehicles depend on speed) [24].

For ranging and measuring the distance from the preceding vehicle different sensors e.g. laser, radar and binocular computer vision are used in ACC. In adverse weather condition, laser based system does not work efficiently. It do not detect and track preceding vehicles. It is mounted on the lower grille and exposed (fairly large box). While radar-based sensor has good results in bad weather conditions. It is small in size and hidden behind bumper. Binocular computer vision system has developed recently. It has front facing video camera which use to obtain depth information about the surrounding, using digital processing.

1.7.2.1 Advantages of ACC

Maintains a safe distance without driver's intervention. Driver has relief in dense traffic from accelerating and braking. It avoids accident as acceleration and braking are done by highly responsive control system. It increase fuel efficiency as vehicle's speed is controlled in a systematic manner.

1.7.2.2 Disadvantages of ACC

Reliable systems are expensive for common use. It causes severe road accident, if system do not response properly as it encourages driver to be careless. It only reacts by looking vehicles in front of it and ignores the traffic signals. In case of platoon, there is string instability.

1.7.3 Cooperative Adaptive Cruise Control

In Cooperative adaptive cruise control (CACC) multiple sensors and systems help the vehicle to follow and control its speed. It uses wireless communication to retrieve more information about its surrounding to decide its control action more efficiently and intelligently.

Different people visualize differently when talking about CACC. The basic concept of each CACC system is to control vehicle with cooperative elements like Infrastructure-to-Vehicle (I2V) and Vehicle-to-Vehicle (V2V) communication [25]. I2V gives information about traffic signals, road signs and traffic farther ahead, where as V2V communication gives information about the other vehicles in the vicinity. Either or both V2V and I2V are used to implement CACC.

Development of CACC is based on two objectives. One is to increase the high way capacity, fuel efficiency and decrease congestion and the second is to improve safety, comfort and customer's convenience and satisfaction. CACC is more attractive than other CC systems for the customers because it has more responsive behaviour to change in velocity of preceding vehicle, providing sense of safety due to its collision warning system [25].

The primary motivation is to reduce the inter-vehicle distance and improve highway capacity. According to Partners for Advanced Transportation and Technology, California (PATH), the inter-vehicle time can be reduce to $0.6s$ using CACC as compare to $1.4s$ through manual driving. PATH shows that with ACC having high penetration, the road capacity do not change too much and has negative effect in the form of platoon due to string instability [26, 27]. With CACC the line capacity can increase from 2200 to 4000 vehicles per hour, with 100% penetration.

The second objective for the development of CACC is the fuel efficiency. On highway fuel consumption is highly influence by the air resistance. Tightly coupled platoon of trucks and passenger vehicles can increase the fuel efficiency [25].

CACC system utilizes I2V communication, which is not focus of this study but it increases the highway capacity and reduces congestion. The most often concepts discussed in I2V communication are variable speed limit and arterial coordinated start [25]. Variable speed limit concept improves the bottle neck capacity by automatically setting the speed limits on upstream. This process reduces the difference of speed and maintains same peak throughout. In CACC the coordinated start helps on traffic signal. When traffic signal turns from red to green, all the vehicles start in coordinated manner and more number of vehicles can pass through a congested intersection as compare to manual driving [25, 28].

1.7.3.1 Length limits for CACC strings

Due to number of reasons, such as performance limitations, safety and integration with unequipped vehicles, there must be a limit in maximum number of vehicles in CACC string. To ensure these criterion one upper limit can be imposed by the range of V2V wireless communication system. Assume all the vehicle in the platoon need to communicate with leader directly, using 5.9 GHz dedicated short range communications (DSRC), which provides at least 300m of the communication range. The length of platoon can be decided based on the distance between the leader and the last vehicle in the platoon [25].

For CACC system where the following vehicles take reference data form leader, transport delays in communication from leader till last vehicle, impose some constraints on string stability but with careful designing of control and actuation system these delays can be minimize enough to allow the length of string up to 15 to 20 vehicles [25]. The most serious limitation arises on the length of CACC string when provide a sufficient gap for lane changing on highway.

When vehicles approach on their destination, CACC string dissolution needs to be careful as much as formation of CACC string. If it is done badly, then it has potential to create a new traffic congestion [28]. Unfortunately, the research on CACC string dissolution strategies is very small. Vehicles coupled with CACC have shorter gap as compared to ACC. It is undesirable to shift entire string from CACC to ACC instantly as it needs larger separations. For efficient dissolving, departing driver need to change the line. The vehicle behind the departing vehicle in the original line becomes leader and reference for other followers in the string. In this way all vehicle leave the string one by one. Each vehicle which leave the string are driven manually [25].

1.7.3.2 Benefits of CACC over ACC

Like human driver, ACC system may not be string stable. Its mean that the oscillations which are produced by accelerating and braking, amplify in upstream direction. This cause phantom traffic jams (best case) or head tail collisions (worst case). In ACC system, it has been shown that, maintaining a fixed headway is not string stable but fix time gap, may or may not be string stable.

In CACC system, this problem is addressed by either improving the stability or by reducing the delay in response by the preceding vehicle. In manual driving this delay depends on the reaction time of driver e.g. leaving throttle and applying brake [29]. In ACC system this delay has reduced, but still there is larger phase delay due to the estimation algorithm required to convert discrete range measurements (taken by lidar or radar) to control signal. In CACC system, each vehicle has information of key parameters such as position, velocity and acceleration of vehicle immediately in-front of it (through sensors), and of leader and other vehicles further in front, using V2V communication. These informations help the vehicle to decide the control action [29].

1.7.3.3 Advantages of CACC

By decreasing gap between vehicles, it increases the lane capacity. It increases fuel efficiency and other environmental benefits.It provides safety of complete traffic system as whole.

1.7.3.4 Disadvantages of CACC

Having few vehicles equipped with CACC system, defeats the purpose. Connected vehicles raise some serious question about privacy and attack by hackers. It will be difficult to determine the liability in case of collision.

1.8 Motivation

In the last century, growth in technology, urbanization and rapid increase of population raised many problems. Trend of personal vehicle has also been increased. There are number of auto-mobile companies around the globe which are increasing the load of vehicles exponentially. Due to these reasons, congestion and traffic jam have been increased especially on highways. In addition, these congestions have harmful effects on environment. The increase in highway intensity leads to road accidents and irregularity in traffic which increase fuel consumption rate.

A lot of research has been done to design an intelligent transport system to address the above problems. The intelligent transport is a system in which vehicles sense their surrounding, take decisions independently and perform control action or help the driver in controlling vehicle. The motivation for designing an intelligent transport system is to increase the convenience of driver in driving task.

A number of vehicle systems have been developed in last few decades which act on different levels. For example line departure warning system, which warns the driver when vehicle starts moving away from the line. One step forward is line keeping system, which applies a torque on the steering wheel, when vehicle leaves the line and bring it back. In urban areas, intelligent transport system has many other applications like parking assist system, intersection collision avoidance and pedestrians detection and avoidance systems.

On highways, to increase the road capacity, vehicles need to be driven close to each other on a safe distance. This thesis focuses on cooperative adaptive cruise control (CACC), which takes data from environment and maintains safe distance from the preceding vehicle. This safe distance may be decided by the designer or can be kept adaptive. This system measure distance from the preceding vehicle through radar and some other sensors and also take information such as position, velocity and acceleration of preceding vehicle through wireless communication. This information helps it to decide the next control action. In the form of platoon, all vehicles are communicating with leader and other vehicles in front of them and maintain smaller headway.

CACC is a convenience system primarily but it raises some serious questions about privacy and attack by hackers. With all this, it has positive effect on safety and increase through put of highway because of smaller distance between vehicles. To take full benefits of this system, more vehicles need to be equipped with CACC. Initially small number of vehicles are in the market equipped with this system but with the passage of time more vehicles enter to the system and gain its benefits.

In literature several techniques has been proposed for CACC. Control techniques which have been used frequently are proportional integral derivative control (PID), model predictive control (MPC) and Pontraygins minimum principle (PMP). In literature, most of the work has been done with the vehicle dynamic model but by adding the engine's states in the system, more real situation can be analyse.

1.9 Thesis Overview

This thesis comprises six chapters including the introduction. In chapter 2, literature review has been given. CACC design is also been explained along the measurements which are required for control purpose. Different platoon configurations are given in this chapter which depend on the available information from the other vehicles in platoon. To control the vehicle in platoon, various control techniques has been used in literature, which are also been explain at the end of this chapter. In chapter 3 complete system model is given. First vehicle kinematics model has been explained which based on relative distance and relative velocity between two vehicle in the platoon. Engine model is also been discussed briefly in this chapter along the conversion of engine's angular acceleration to vehicle linear acceleration. In Chapter 3 problem statement is given along the objective of this work. Different optimization techniques have been discussed in chapter 4. This chapter gives a brief overview of optimal control theory and comparison between the PMP and dynamic programming. Chapter 5 is the main part of this thesis in which the proposed technique has been applied on the model. Initially the cost formulation for the system is given which depends on design objective. Then the optimal control input for the system has been designed using Pontraygin's minimum principle. To solve the optimal problem a numerical algorithm has also been presented in this chapter. Finally the simulation results are given at the end of this chapter which satisfy the design objective. Conclusion has been drawn along the future work in chapter 6.

I feel that in entirety, the subject is extremely beautiful with great potential for further research . Hence, in this thesis I share not my knowledge or my work, but the enthusiasm and the passion that I developed along the way.

Chapter 2

Literature Review

In literature, several control techniques have been defined for CACC system. In each of these techniques different constraints and limitations have been considered to obtain control objective. Most of them are only tested on simulation and rarely have been implemented as prototype. In this chapter different CACC platoon configurations along with the commonly used control techniques have been presented.

2.1 CACC Design

The task of CACC is to ensure that the vehicle follows its preceder at a safe distance and travels at a predetermined speed in the absence of a disturbing predecessor [30]. During this task CACC should be able to guarantee the string stability. A platoon is string stable, if the disturbance disappears during propagation to the end of the string [25, 26]. CACC should also ensure that the (variations in) acceleration and deceleration remain in a comfort zone. In addition, frequent switching between acceleration and braking should be avoided [31]. The block diagram of vehicle having CACC has been shown in figure 2.1. CACC controls the position, speed and acceleration of the vehicle. Its aim is to make the speed, position and acceleration error to zero. The reference signal in the figure 2.1 contains the desired position, velocity and acceleration. The desired speed is often the speed of the predecessor [25].

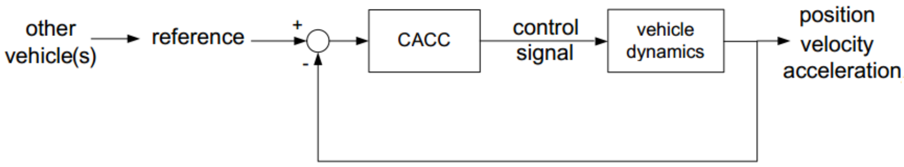

FIGURE 2.1: CACC design [32]

If there is no preceder or it is moving with speed higher than the pre-set speed of follower, then vehicle will move at its pre-set speed. The desired position is usually derived from the position preceder. If there is no vehicle infront, then there is no reference position, and only speed and acceleration is needed to be controlled [28, 31].

2.1.1 Measurements

The CACC system has information about the position, speed and acceleration of ego(self) vehicle and need to know about the preceder. Sometimes radar sensors are used to measure the distance, which are fast and provide robust performance in different weather conditions but these systems are pretty expensive. An alternative is light detection and ranging (LIDAR), in which the laser pulses are used for ranging and finding the speed difference with the first preceder. They are cheap but have reduced visibility in fog or smoke. Some CACC systems use global positioning system (GPS) to find the location [25, 33].

The position accuracy of the GPS is $10 - 20m$ and can be improved to $1 - 2cm$ by using the additional differential positioning error correction signals, from the nearest ground GPS base station. These FM signals are received by a vehicle-mounted radio receiver.The update frequency of this differential GPS is too low for the ACC ($1Hz$) [34].

In addition, differential GPS signals can be blocked by buildings, trees and bridges, and not always be available. To have position information available at all time, the differential GPS data can be merged with data from dead reckoning sensors, such as wheel encoders and accelerometers. This data fusion can be performed by an advanced Kalman filter which estimates the parameters for the moment, when no new GPS signal is received, based on information from navigation sensors billing [34]. While using this GPS method, the states of the preceding vehicle are not measured, but they can be obtained by vehicle-to-vehicle communication. From the position data (in time) obtained by the above detection methods, the speed and acceleration can easily be calculated. Vehicles also use wireless communication to inform the other vehicles about their condition and possibly send information about their next expected status e.g. acceleration and braking [35].

2.2 Platoon Formation

In the case of platoon, the position of vehicles are numbered starting with the leader as vehicle 1, his successor as vehicle 2 and so on as shown in figure 2.2. If CACC is being designed for vehicle i its predecessor is vehicle i 1, counting all the way further up to vehicle 1, which is the platoon leader in case of a platoon [32, 36].

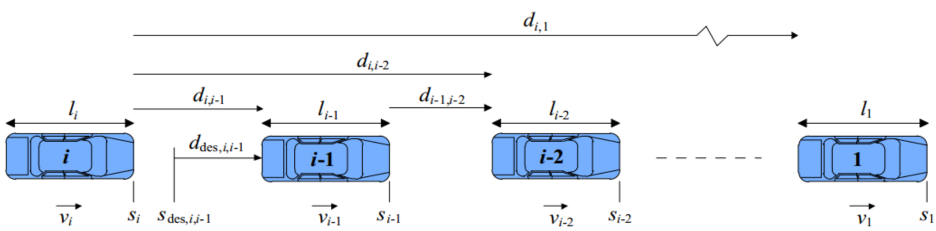

FIGURE 2.2: Platoon formation [32]

2.2.1 Platoon Configuration

Depending on the avalible information from the vehicles in platoon, there are three different designs of CACC. These designs depend on informaiton collection configuration [32].

Configuration I

Only communicated data from the direct preceder is used. So vehicle i uses data from vehicle $i-1$ only.

Configuration II

Communicated data from the first two preceder is used. So vehicle i uses data from vehicles $i-1$ and $i-2$.

Configuration III

Communicated data from the direct preceder and platoon leader is used. So vehicle i uses data from vehicles $i-1$ and 1.

2.3 CACC with PID

Proportional Integral Derivative (PID) controller is reliable and have practical control solution for many industrial processes. One of the main advantage of PID controller is their ease of implementation and customization, which provides a good compromise between simplicity and the cost of implementation [32]. Current research ensures reasonable stability margins and good overall performance for a variety of processes [17].

Although PID-based CACC controllers are safe for chord (osilation) scenarios, but they allow two vehicles to crash during validation. The PID itself can not predict the dangerous situations. The first vehicle in platoon has the greatest risk of collision with its predecessor. Its followers are less at risk because they use similar control actions, especially when using information from multiple preceders [32].

It is easy to design a CACC controller that achieves smooth throttle/brake paths with PID, but at the expense of a slower response. However, it should be noted that acceleration gain remain negligible after tuning (too small). This also results in a slow response which, in turn, requires a large headway for predecessor and infact this vehicle could not avoid an accident during the validation scenario [37].

In [38] a common types of attack injected on the application layer of connected vehicles to show their vulnerability in comparison to autonomous vehicles. It also proposed a decision support system that eliminates risk of inaccurate information. The microscopic work simulates a CACC system with a bi-objective PID controller and a fuzzy detector. A case study is illustrated in detail to verify the system functionality.

Introducing PID controller into CACC system, there still exist some problems, for example, research on the driving comfort and fuel consumption is far from enough, as well as could not rationally use the intelligent algorithm to tune PID controller. In order to solve the problems above [39] presents a method to tune PID controller using chaotic ant swarm (CAS) to make the PID controller better match the safe distance model.

2.4 CACC with MPC

CACC based on MPC is safer than with PID and therefore preferred as a control method for CACC as it can anticipate the dangerous situations aswell. MPC-based controllers kept the vehicles safe at relatively small distance in platoon. In MPC-based controllers the peaks and acceleration oscillations may flactulate from front to back, but do not necessarily grow. Therefore, it is expected that MPC-based CACC can be safe even for larger platoon. Because safety is more important than comfort, so it can be concluded that the MPC is actually superior to the PID as a control method for CCAP [32].

In CACC based on MPC, it is preferable, with the current states of the direct preceder, to have at least the current states of the second preceder and/or the expected future states of the direct preceder in order to improve the stability of the strings. This means that better string stability can be achieved, if the CACC controller knows about next state of direct predecessor [33]. Here, the current state of the second predecessor indirectly indicates the next state of direct predecessor.

It is not easy to develop a CACC controller that achieves smooth throttle/braking with MPC. It is very important to tune the weights which punished the increment in throttle/brake. The results of the case study have shown [32] that too low value for this weight can cause sudden spikes and vibrations in the MPC acceleration. Because large weights can cause other problems (e.g. low responsiveness to brake of preceder, later on need excessive breaking). To find the right value of weight is a difficult task.

In [40] a linear model predictive control approach to Cooperative Adaptive Cruise Control is presented, directly minimizing the fuel consumption rather than the acceleration of the vehicle. To this end the nonlinear static fuel consumption map of the internal combustion engine is included into the control design by a piecewise quadratic approximation. Inclusion of a linear spacing policy prevents rear end collisions. Simulation results demonstrate the fuel and road capacity benefits, for a single vehicle and for a string of vehicles, equipped with the proposed control, in comparison to vehicles operated by a non-cooperative adaptive cruise control.

A MPC based approach to improve a recently developed class of CACC schemes is presented in [37]. The PID structure used previously is replaced with MPC, which is able to accommodate actuator limits and parameter estimation. In addition to the regular CACC functionalities, rear end collision control is also incorporated. This approach is able to avoid rear end collisions with the following car, as long as it can still maintain the safe distance with the preceding vehicle.

2.5 CACC with PMP

An optimal control framework for the modelling of driver assistance systems is proposed in [41]. In this approach, the acceleration of the leading vehicle is controlled by the driver and all other vehicles in platoon follow it by optimizing a generic objective/cost function. To solve the problem of optimization, a numerical approach for pontryagin's minimum principle has been proposed in [41]. Competitive and collaborative controllers are proposed in which each vehicle optimizes its own situation or all vehicles work together to optimize the overall performance of the platoon. The results show that the computational complexities of the proposed approach are small enough to enable real-time computations of auto-tracking strategies, as compared to previous approaches. An important feature is that,these complexities do not increase exponentially with either by increasing the complexity of predictive model or the size of the control vector [42].

In addition, the example of joined control for CACC has proved that the cooperative driving strategy can improve overall performance of the platoon as compare to competitive driving strategy. In particular, computational complexities have slightly been worsened in cost of joint control for multiple vehicls, as compared to single vehicle control [41].

2.6 Gap Analysis

In literature different control techniques have been used for cooperative adaptive cruise control, and some of them are given above. All these techniques gave good results and some of them have been used for practical applications as well. But while applying these techniques, most of the time only a simple kinematic model of vehicle is used, which contains the acceleration of leader and follower.

2.7 Chapter Summary

When vehicles are moving in the form of platoon, they need to have small and safe inter vehicles distance. To ensure the string stability in the platoon, CACC system has been defined in literature. For CACC design, states e.g. position, velocity and acceleration of the leader and other vehicles in the platoon are required. Each vehicle in the platoon can be configure in three different configurations, depending on the information available. To optimize the distance between the vehicles, different control techniques has been discussed in the literature. PID is used more widely for simulation as well as implementation. These techniques used different constraints to optimize safe distance between vehicles.

In CACC design, engine model has not been considered in system. In next chapter vehicle kinamatic model having engine states has been presented along the problem statement.

Chapter 3

System Modelling and Problem Description

In this chapter, first vehicle kinematic model is described, then to find the acceleration of vehicle, engine model has been discussed briefly. The complete model of system is obtained by combining the engine states and kinematic model of vehicle. Finally, the control problem has been defined.

3.1 Vehicle's State Prediction Model

Consider the state prediction model, the follower is on a certain distance from the leader, as shown is figure 3.1. The desired states are headway (distance between the leader and follower) s_i and the relative velocity Δv_i between leader and follower.

Dynamics for the vehicle i at time t, can be defined by the following kinematic equations [41]:

$$\frac{d}{dt}x_p = \frac{d}{dt}\begin{pmatrix} s_i \\ \Delta v_i \end{pmatrix} = \begin{pmatrix} \Delta v_i \\ a_{i-1} - a_i \end{pmatrix} \tag{3.1}$$

With initial conditions:

$$s_i(t_k) = r_{i-1}(t_k) - r_i(t_k) \tag{3.2}$$

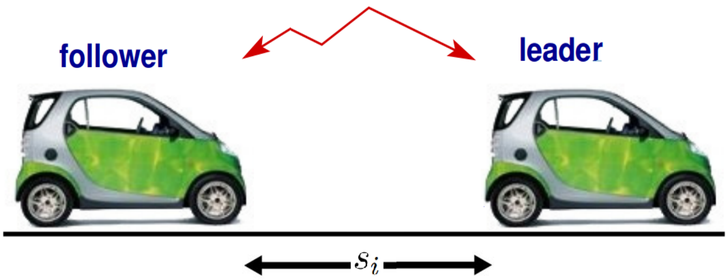

FIGURE 3.1: Vehicle state model [41]

and

$$\Delta v_i(t_k) = v_{i-1}(t_k) - v_i(t_k) \tag{3.3}$$

where a_i, v_i and r_i denote the acceleration, velocity and position of vehicle i, respectively. Moreover a_{i-1}, v_{i-1} and r_{i-1} are the acceleration, velocity and position of predecessor, respectively.

For the simple case of leader and follower, a_{i-1} is the acceleration of leader which is received through some communication link and a_i is the acceleration of follower is computed from the engine model as given in equation 3.6.

3.1.1 Engine Model

Engine is the device which convert chemical energy into mechanical energy. Fuel is first converted to thermal energy by the means of combustion inside the cylinder, which results in the rise of temperature and pressure. This high pressure gas expands against the piston in cylinder. Piston causes to and fro movement which is converted into circular motion by using crankshaft.

In this work mean value engine model (MVEM) is used. This model is based on angular speed, ω and manifold pressure, P_m as states of engine, and throttle angle, α as input [43].

With some assumptions, MVEM can be defined as follows:

$$\dot{P}_m = A_1 \left(1 - e^{\frac{P_m}{P_a}-1} - \cos(\alpha) + \cos(\alpha)e^{\frac{P_m}{P_a}-1}\right) - A_2 P_m \omega \tag{3.4}$$

$$\dot{\omega} = B_1 P_m - B_2\omega - B_3\omega^2 - T_l \tag{3.5}$$

$$A_1 = \left(\frac{RT_m P_a C_d}{V_m}\right)\left(\sqrt{\frac{2\gamma}{(\gamma-1)RT_a}}\right)\left(\frac{\pi D^2}{4}\right)$$

$$A_2 = \left(\frac{30V_d\eta_v}{\pi V_m}\right)$$

$$B_1 = \left(\frac{V_m}{4\pi J_e}\right)\left(1 - \frac{1}{c_r^{\gamma-1}}\right)\left(\frac{(c_r^{2-\gamma})(c_r^{\gamma-1}-1)(\eta_c Q)}{(\gamma-1)(c_r-1)(C_v T_m AFR)}\right)$$

$$B_2 = \left(\frac{V_m}{4\pi J_e}\right)$$

$$B_3 = \frac{0.05V_d\pi}{18 \times 10^4 J_e}$$

The model parameters have been defined in table 3.1.

3.1.2 Vehicle Acceleration

Engine's angular acceleration ($\dot{\omega}$) given in equation 3.5, will be used to calculate the acceleration of (following) vehicle with the help equation 3.6.

$$a_F = \frac{\dot{\omega}\ Td}{336.13\ Ar\ Tr} \tag{3.6}$$

The above parameters have been defined in table 3.2.

The acceleration of follower is calculated in equation 3.6 will be use to find the state prediction model.

TABLE 3.1: MVEM parameter description, nominal values and units

Symbol	**Description**	**Values/Units**
P_a	Ambient pressure	$101325P_a$
T_m	Manifold temperature	325 K
T_a	Ambient temperature	298
α_{cl}	Throttle angle at closed position	9.8
D	Inlet Diameter	0.054 m
R	Specific gas constant	287 J/kg.K
CD	Throttle discharge coefficient	0.8
γ	Ratio of heat capacities	1.4
V_d	Displaced volume	$0.001294m^3$
V_m	Manifold volume	$0.001127m^3$
η_{vol}	Volumetric efficiency	0.7
η_c	Combustion efficiency	0.9
AFR	Air to fuel ratio	14.7
J_e	Engine inertia	$0.25kg.m^2$
Q	Heat value of fuel	$44kJ/Kg$
c_v	Heat capacity at specific volume	$717J/(m^3K)$
c_r	Compression ratio	10
T_l	Load torque	$30N$

TABLE 3.2: Vehicle's acceleration conversion parameters

Symbol	**Description**	**Values/Units**
Ar	Axle Ratio	0.8
Tr	Transmission rate	3.73
Td	Tire diameter	$24in$

3.2 System Model

In this section, overall model has been defined by joining two states, distance headway s_i and relative velocity ΔV_i of vehicle model and engine's states, manifold pressure P_m and angular speed ω.

Let

$$
\begin{aligned}
x_1 &= s_i \\
x_2 &= \Delta v_i \\
x_3 &= P_m \\
x_4 &= \omega
\end{aligned}
\tag{3.7}
$$

For calculation and simulation simplicity assume:

$$cos(\alpha) = u$$

The overall model of any vehicle i can be defined as:

$$
f(t, x, \alpha) = \dot{x} = \begin{pmatrix} x_2 \\ a_L - \dfrac{x_4}{\psi} \\ A_1 \left(1 - e^{\frac{x_3}{P_a} - 1} - u + u e^{\frac{x_3}{P_a} - 1}\right) - A_2 x_3 x_4 \\ B_1 x_3 - B_2 x_4 - B_3 x_4^2 - T_l \end{pmatrix}
\tag{3.8}
$$

where a_L is the acceleration of leader and the control input for the system is α. As throttle angle α for engine is from 9.8^o to 85^o, so $cos(\alpha)$ remains between 0.087 to 0.985 and there a bound, $0.087 < u < 0.985$.

3.3 Problem Description

In daily routine, every one needs to deal with high traffic on highway which causes congestion, accidents and other undesirable (environmental and financial) loses. A lot of research has been going on to find comfortable and safe ways of transportation with high throughput. To increase throughput by decreasing the distance between vehicles, different speed controlling techniques (e.g. ACC, CACC) have been used.

CACC system is the one which has widely been introduced in vehicles to increase the throughput of highway by minimizing the distance between vehicles. It allows shorter inter vehicle distance as compared to manual driving. In literature different techniques are found which minimize the distance between vehicles with safety and comfort.

The control objective is to bring the headway on a desired value and relative velocity around zero.

3.4 Chapter Summary

In this chapter, system model has been explained in detail. For designing CACC a simple vehicle kinematics model has been used so far which based on the vehicle's states e.g. position, velocity and acceleration. In the given model, we introduced the engine's dynamics in the system. Now, instead of designing the acceleration of vehicle, we design the input for engine. Vehicle acceleration is then calculated from the angular acceleration of engine. A complete model has been defined in this chapter, which based on two states from vehicle kinematics model and two from engine dynamics model. At the end, the problem description has been defined.

As the control problem is to maintain a fix distance between the vehicles. Different techniques have been used to obtain a fix distance between the vehicle. Before applying the optimization technique, a brief overview of optimal control theory has been given in succeeding chapter.

Chapter 4

Optimal Control: An Overview

In order to understand the optimal control with better comprehension and insight knowledge, its preferable to look the entire spectrum of optimal control. In this chapter a brief overview of optimal control has been given.

There are two basic ideas, dynamics programming and Pontraygin's minimum principle(PMP) with allied optimal control principles on which the optimal theory has been established [44]. Pontryagin along his compatriots laid the foundation of PMP. Richard Bellman had developed the parallel approach for optimal control independently, later it was known as dynamic programming (DP). DP based on optimal principle while PMP on variational approach [45].

4.1 Pontraygin's Minimum Principle

Pontraygin's minimum principle is the generalization of Euler-Lagrange equations which also have constraints on control input. Here a special case for the Pontraygin's minimum principle is given but it covers a large group of control problems. Lets assume a performance index function which needs to minimize:

$$J = \phi\left(x(t_f)\right) + \int_{t_0}^{t_f} L\left(x(t), u(t)\right) dt \tag{4.1}$$

where

$$\dot{x} = f(x, u) \tag{4.2}$$

with initial and final conditions:

$$x(t_0) = x_0, \text{ and } x(t_f) = x_f$$

Consider the Hamiltonian function:

$$H(x, u, \lambda) = L + \lambda^T f \tag{4.3}$$

where λ is co-state vector haveing fixed final time, t_f. By using the Luenderger's development [46], adjoin a term which sum up to zero with J. As the equation $\dot{x} = f(x, u)$ must satisfy the state trajectories, so:

$$f(x, u) - \dot{x} = 0 \tag{4.4}$$

Modify the objective function as:

$$\tilde{J} = J + \int_{t_0}^{t_f} \lambda(t)^T \left(f(x, u) - \dot{x}\right) dt \tag{4.5}$$

Because equation 4.4 satisfies all state trajectories, for any choice of λ, the value of J and the $\tilde{J}$ is same.

$$\begin{aligned} \tilde{J} &= \phi\left(x(t_f)\right) + \int_{t_0}^{t_f} L\left(x(t), u(t)\right) dt + \int_{t_0}^{t_f} \lambda(t)^T \left(f(x, u) - \dot{x}\right) dt \\ &= \phi\left(x(t_f)\right) + \int_{t_0}^{t_f} F\left(H(x, u, \lambda) - \lambda^T \dot{x}\right) dt \end{aligned} \tag{4.6}$$

Let $u(t)$ is the nominal control strategy which gives state trajectory $x(t)$. If there is another control strategy $v(t)$ which is close to $u(t)$, then it will give a new state trajectory. This state trajectory is the perturbed version of nominal state trajectory $x(t)$ and can be shown as:

$$x(t) + \delta x(t)$$

The performance index function $\tilde{J}$ will also change, with the change in state trajectory.

$$\delta\tilde{J} = \phi\left(x(t_f) + \delta x(t_f)\right) - \phi\left(x(t_f)\right) + \int_{t_0}^{t_f} \left(H(x+\delta x, v, \lambda) - H(x, u, \lambda) - \lambda^T \delta\dot{x}\right) dt \tag{4.7}$$

Solution of above integral by using integration by part is:

$$\int_{t_0}^{t_f} \lambda^T \delta\dot{x} dt = \lambda(t_f)^T \delta x(t_f) - \lambda(t_0)^T \delta x(t_0) - \int_{t_0}^{t_f} \dot{\lambda}^T \delta x dt \tag{4.8}$$

As changing the control strategy, there is no change in initial condition, so $\delta x(t_0) = 0$. Take this in equation 4.7:

$$\begin{aligned}\delta\tilde{J} &= \phi\left(x(t_f) + \delta x(t_f)\right) - \phi\left(x(t_f)\right) - \lambda(t_f)^T \delta x(t_f) \\ &+ \int_{t_0}^{t_f} \Big(H(x+\delta x, v, \lambda) - H(x, u, \lambda) + \dot{\lambda}^T \delta x\Big) dt\end{aligned} \tag{4.9}$$

Use first order approximation of $\phi\left(x(t_f) + \delta x(t_f)\right) - \phi\left(x(t_f)\right)$, and, subtract and add $H(x, v, \lambda)$ in the integral

$$\begin{aligned}\delta\tilde{J} &= \left(\nabla_x \phi|_{t=t_f} - \lambda(t_f)\right)^T \delta x(t_f) \\ &+ \int_{t_0}^{t_f} \Big(H(x+\delta x, v, \lambda) - H(x, v, \lambda) + H(x, v, \lambda) - H(x, u, \lambda) + \dot{\lambda}^T \delta x\Big) dt\end{aligned}$$

Now replace $\Big(H(x+\delta x, v, \lambda) - (H(x, v, \lambda)\Big)$ with first order approximation:

$$H(x+\delta x, v, \lambda) - H(x, v, \lambda) = \frac{\partial H}{\partial x}\delta x,$$

gives

$$\begin{aligned}\delta\tilde{J} &= \left(\nabla_x \phi|_{t=t_f} - \lambda(t_f)\right)^T \delta x(t_f) \\ &+ \int_{t_0}^{t_f} \left(\left(\frac{\partial H}{\partial x} + \dot{\lambda}^T\right) \delta x + H(x, v, \lambda) - H(x, u, \lambda) + \dot{\lambda}^T \delta x\right) dt\end{aligned}$$

Selecting the λ as solution for above differential equation

$$\frac{\partial H}{\partial x} + \dot{\lambda}^T = 0$$

where the final condition is

$$\lambda(t_f) = (\nabla_x \phi|_{t=t_f})$$

Finally the $\delta \tilde{J}$ is:

$$\delta \tilde{J} = \int_{t_0}^{t_f} (H(x, v, \lambda) - (H(x, u, \lambda))\, dt$$

If $u(t)$ is optimal control then we must have $\delta \tilde{J} \geq 0$, so $H(x, v, \lambda) \geq H(x, u, \lambda)$. Following theorem summarized the development.

Theorem: Necessary conditions for $u \in U$ to minimize equation 4.2 subject to 4.1 are:

$$\dot{\lambda} = -\left(\frac{\partial H}{\partial x}\right)^T$$
$$H(x^*, u^*, \lambda^*) = min H(x, u, \lambda)$$

Where $H = L(x, u) + \lambda^T f(x, u)$ and λ is the co-state dynamics.

4.1.1 Linear Quadratic Control

Rudolf E. Kalman and his colleagues from United States published three important articles between 1960 to 1961. Design equation for linear quadratic control(LQC) has given by Kalman in one of these articles. When the cost function is quadratic and system is linear then it can be considered the special case of PMP [44].

Consider, the state space of a system as bellow:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4.10}$$
$$x(t_0) = x_0, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m$$

here B is the input matrix and A is the system matrix. The performance index for the Linear Quadratic problem can be defined as follows:

$$J = \frac{1}{2}x^T Sx + \frac{1}{2}\int_{t_0}^{t_1} \left(x^T Qx + t^T Ru\right) dt \tag{4.11}$$

where t_0, t_1 are the starting and end times, S and Q are the semi-definite positive matrix and R is the positive definite matrix. In the case of an infinite horizontal problem, i.e. $(t_1 \to \infty)$, the matrix $S = 0$. It is assumed that there exist $u = -Kx$ a stabilizing feedback control that minimizes the cost of the function J. Such feedback control exists if the Algebraic Riccati equation has solution for positive definite P.

$$Q + A^T P + +PA - PBR^{-1}B^T P = 0$$

The controller gain K can be defined as bellow:

$$K = R^{-1}B^T P$$

This stabilizing controller is linear quadratic regulator (LQR).

4.1.2 Optimal Estimation and Kalman Filter

The concept of estimation theory and optimal filtering along the design procedure and mathematics for discrete Kalman filter has discussed in third paper of Kalman, mentioned in previous paragraph [47]. Later on in 1961 Kalman and Bucy proposed the continuous counterpart of discrete kalman filter(DKF). It is also known as linear quadratic estimator. In Kalman filter a series of states values measured from sensors taken as input and perform recursive algorithm and estimate the current state. This is usually used in state feedback control, where all the states are unavailable or not feasible to measure. Stratonovich-Kalman-Bucy filter is consider as a special case as it gives more general, non-linear filter which is proposed by the Soviet mathematician, Ruslan L. Stratonovich previously [44].

The strength of the Kalman filter is that, it can forecast past, presence and future. The algorithm is based on a cyclic process having two processes, namely prediction or update, and correct or measurement update.

Filter estimate the state based on the previous estimation, without taking data from sensors in update sub-process. This is called a priori estimation. Then these priori estimated states and states measured by the sensors both are used to find the corrected estimation in correct sub-process. This estimation is called posteriori estimation. Therefore the measurement update acts like a feedback in filtering. Equations are given briefly to describe the filter below [47].

Lets take a time-invariant linear discrete time control system:

$$
\begin{aligned}
x_k &= Ax_{k-1} + Bu_{k-1} + w_{k-1}, \quad && x_k \; \epsilon \; \mathbb{R}^n \\
y_k &= x_k + v_k, && y_k \; \epsilon \; \mathbb{R}^m
\end{aligned}
$$

where v_k and w_k represent measurement noise and process respectively supposed to be white and Gaussian. Let $\bar{a}_k$ a priori and $\hat{a}_k$ be posteriori state estimation. State estimation are as follows:

$$\hat{x}_k = \bar{x}_k + K(y_k - C\bar{x}_k)$$

Intuitively this means that we can make the priori estimation of state $\bar{x}_k$ and then calculate the posterior estimation by adding the correction term which is based on scaled $(y_k - C\bar{x}_k)$. Filter gain K is a $n \times m$ matrix. To calculate K, consider the error in a priori and posterior state as:

$$
\begin{aligned}
\bar{e}_k &= x_k - \bar{x}_k \\
\hat{e}_k &= x_k - \hat{x}_k
\end{aligned}
$$

Error co-variance matrices are:

$$\bar{P}_k = E\left[\bar{e}_k \bar{e}_k^T\right]$$
$$\hat{P}_k = E\left[\hat{e}_k \hat{e}_k^T\right]$$

The gain K of the filter is set as the posteriori covariance $\hat{P}_k$ should be minimize. It is derived from another Riccati equation, which is not given here and beyond the scope of this document. Covariance can be minimize by the following K:

$$\begin{aligned} K_k &= \bar{P}_k C^T \left(C\bar{P}_k C_T + R\right)^{-1} \\ &= \frac{\bar{P}_k C^T}{C\bar{P}_k C_T + R} \end{aligned}$$

where R is the covariance matrix of noise. A similar result applies to continuous-time systems and is called the Kalman-Bucy filter. The Kalman filter is an exceptionally powerful estimator, but only for linear systems. Most of Kalman's filters are successful for non-linear applications. For this purpose, the system is linearised about the current state with current covariance and mean. This filter is called Extended Kalman Filter (EKF).

4.1.3 Duality between Optimal Estimation and Kalman Filter

Together, the Linear Quadratic Regulator and Kalman filters address the linear-Gaussian square (LQG) problem. Kalman described that the linear quadratic regulator is dual problem with Kalman filter as one can be defined in-terms of other [48]. This can be determined by looking at the Riccati equation for controller and regulator given in the standard document.

4.2 Dynamic Programming

Richard Bellman introduced dynamic programming in 1957 as an alternative to optimal control technique [49]. The word programming has no connection to software programming but now it also use to optimize the algorithem in software engineering domain. Dynamic means the system having evolving nature and programming means to plan. So the true meaning of dynamic programming is to plan an optimal action for the systems whose states change with time. Dynamic programming is a discrete optimization method based on Bellman's optimization principle, which simply states that a part in the optimal state trajectory is itself optimal. It solves the optimal problem in reverse i.e. it takes the terminal state and solve in backward and minimize the cost. The optimal path with optimal control and allied cost is identified when state reaches its initial value. $v(x)$ is the optimal value function which takes the system from initial state x_0 to final state x_f. For more detail one can go through [50].

4.3 Hamilton Jacobi Bellman Equations

In the continuous time domain, the solution of optimal control lead towards very complex partial differential equation which is based on work of Carl Jakob Jacobi and W. R. Hamilton known as Hamliton-Jacobi-Bellman equation [51]. Consider a system whose dynamics and cost are given as below:

$$\begin{aligned} \dot{x} &= f(t, x, u) \\ J &= \int_{t_0}^{t_f} L(t, x, u)dt + K\left(x(t_f)\right) \end{aligned}$$

where t_0 is the initial time, t_f is the final time, L is running cost and K is the terminal cost linked with terminal state x_f. The final state of HJB equation can be define as:

$$0 = \frac{\partial}{\partial t} J^* + min \left\{ L(t, x) + (\frac{\partial}{\partial t} J^*, f(t, x, u)) \right\}$$

This is partial differential equation in x and t, the famous Hamilton Jacobi Belman equation. The solution of this HJB gives optimal control [44].

4.4 Comparison between PMP and HJB Equation

The comparison of PMP and HJD equation is given in table 4.1

TABLE 4.1: Comparison between PMP and HJB Equation

PMP	**HJBE**
Gives the optimal conditions for the first order	Sufficient conditions are given for optimality
The resultant is first order differential equation, which is comparatively easy to solve	It gives partial differential equations in time t
Optimal trajectories have been found for candidates	At start, the value function is known which is unusual
It has geometrical approach so gives better understanding of system dynamics	Use as feedback control
Generally its analytical solution is possible	On increasing the system's states, complexity increases exponentially
Gives open loop control results	HJB equations usually have non-smooth solution, but for generalized solution, different theories have defined in literature

Therefore, the use of PMP is generally much easier than using HJB. If someone successfully solves HJB then for optimal control problem, it will be the best solution. But it is very rare to find solution.

4.5 Chapter Summary

In this chapter the notion of optimal control has been established and competing directions have been explored. With the discovery of PMP, optimal control theory reached to its zenith. HJB equations and PMP are two parallel optimal control approaches with peculiarities related to them. PMP with its geometric view not only solves the optimal control problem but it also gives more insight to the behaviour of the system. With motivation for PMP constituted, we are now in a position to use it on system given in chapter 3, which is the main goal of this thesis. Controller designing has been given with algorithm for numerical solution and results are presented in the next chapter.

Chapter 5

Controller Design and Results

In cooperative adaptive cruise control (CACC) vehicles communicate to each other and decide their control action. This chapter covers the details of the controller designing. Pontraygin's minimum principle (PMP) is used to design the controller for the following vehicle. An algorithm for numerical solution is also presented. The chapter also includes the simulation results of model discussed in chapter 03.

5.1 Cost Formulation

In optimization, the cost function maps an event, variables or states on a real number representing cost related to that event. An optimization problem is to minimize the cost function.

Desired cost function J for the car following task:

$$J(t,x,u) = \int_{t_0}^{t_f} L(t,x,u)dt + \phi(t_f, x(t_f)) \tag{5.1}$$

where x denotes the states of vehicle i, s^* is desired distance head way, u is controlled input and t_f is prediction horizon. In cost function L is so-called running cost during the infinitesimal time period $[t, t + dt)$. ϕ is so-called terminal cost which describes the remaining cost of system at the end of prediction horizon t_f.

In the given system, two states are coming directly from the engine of vehicle i and other two can be predicted. Assume that the follower predicts the dynamics of the vehicle by using the speed profile of leader. This assumption is not surely accurate but the controlled vehicle (follower) will response according to this expected profile. Assume there are only two vehicles in the platoon(leader and follower).

As the control objective is to maintain a fix relative distance between two vehicles. For this purpose the relative speed between two vehicles need to be zero after attaining the desired headway. So we take relative distance and relative velocity in cost function along the control input.

The running cost for the system can be defined as [41]:

$$L = \frac{1}{2}u^2 + \frac{\beta_1}{2}(x_1 - s^*)^2 + \frac{\beta_2}{2}x_2^2 \tag{5.2}$$

where x_1 is relative distance s_i, x_2 is relative velocity ΔV, u is controlled input and s^* is desired distance headway. For simplification choose the terminal cost $\phi = 0$.

The running cost has three cost components and trade off exists between them. The controller aim is to maintain the headway to a specified distance s^*, minimize the relative velocity and control input. Weights of the cost function are described by β_1 and β_2 respectively, which need to bear, if not driving at specific distance headway and having non-zero relative velocity.

5.2 Controller Design

To solve the system given in Chapter 3 using PMP optimal control define an Hamiltonian based on system states, co-states and running cost of system.

$$H(t, x, u, \lambda) = L(t, x, u) + \lambda' f(t, x, u) \tag{5.3}$$

where λ' defines the co-states or marginal cost of x. These show the extra cost in J due to small change δx in state x.

For optimal control u^*, the necessary conditions can be defined by using the Hamiltonian:

$$H(t, x, u^*, \lambda) \leq H(t, x, u, \lambda), \forall u \tag{5.4}$$

The co-state dynamics determined as follow:

$$-\frac{d}{dt}\lambda = \frac{\partial H}{\partial x} = \frac{\partial L}{\partial x} + \lambda\frac{\partial f}{\partial x} \tag{5.5}$$

Subject to final conditions at end of control horizon $t = t_f$ (where starting time is $t = 0$):

$$\lambda(t_f) = \frac{\partial \phi}{\partial x}(t_f, x(t_f)) \tag{5.6}$$

The Hamiltonian for the given system is:

$$\begin{aligned} H = {} & \frac{1}{2}u^2 + \frac{\beta_1}{2}(x_1 - s^*)^2 + \frac{\beta_2}{2}x_2^2 + \lambda_1 x_2 + \lambda_2\left(a_L - \frac{B_1x_3 - B_2x_4 - B_3x_4^2 - T_l}{\psi}\right) \\ & + \lambda_3 A_1\left(1 - e^{(\frac{x_3}{P_a}-1)} - u + ue^{(\frac{x_3}{P_a}-1)}\right) - \lambda_3 A_2 x_3 x_4 + \lambda_4\left(B_1x_3 - B_2x_4\right. \\ & \left. - B_3x_4^2 - T_l\right) \end{aligned} \tag{5.7}$$

The co-state equation can be defined as below:

$$-\dot{\lambda}_1 = \frac{\partial H}{\partial x_1} = \beta_1\left(x_1 - s^*\right) \tag{5.8}$$

$$-\dot{\lambda}_2 = \frac{\partial H}{\partial x_2} = \beta_2 x_2 + \lambda_1 \tag{5.9}$$

$$-\dot{\lambda}_3 = \frac{\partial H}{\partial x_3} = -\lambda_2\frac{\beta_1}{\psi} + \lambda_3\frac{A_1}{P_a}\left(-e^{(\frac{x_3}{P_a}-1)} + ue^{(\frac{x_3}{P_a}-1)}\right) - \lambda_3 A_2 x_4 + \lambda_4 B_1 \tag{5.10}$$

$$-\dot{\lambda}_4 = \frac{\partial H}{\partial x_4} = \frac{\lambda_2}{\psi}\left(B_2 + 2B_3x_4\right) - \lambda_3 A_2 x_3 + \lambda_4\left(-B_2 - 2B_3x_4\right) \tag{5.11}$$

To find the optimal control input, take derivative of H with respect to u and put equal to zero:

$$\frac{\partial H}{\partial u} = u + \lambda_3 A_1 \left(-1 + e^{(\frac{x_3}{P_a}-1)}\right) = 0$$

$$u_{opt} = \lambda_3 A_1 \left(1 - e^{(\frac{x_3}{P_a}-1)}\right) \tag{5.12}$$

5.3 Algorithm for Numerical Solution

The most difficult task in PMP is to find its numerical solution. The boundary conditions are defined on two endpoints. For the states x, initial conditions $x(0)$ are given and for co-states λ, terminal conditions $\lambda(t_f)$ are known. These boundary values make the solution of differential equation difficult.

To address this problem, solve the state equations x, forward in time and co-state dynamics λ backward in time. The algorithm is summarized as follow [13]:

Algorithm 1 Algorithm for numerical solution of PMP

1: Choose learning rate $0 < \alpha < 1$, set iteration no. $n = 1$ and the stopping criteria ϵ_{max}.
2: Set $\Lambda^0(t) = 0$, for time interval $0 \leq t \leq t_f$.
3: Solve the state equations as:

$$\frac{d}{dt}x^{(n)} = f\left(t, x^{(n)}, u^*(t, x^{(n)}, \Lambda^{(n-1)})\right) \tag{5.13}$$

with $x^{(n)}(0) = x_0$ forward in time.
4: solve the co-state dynamics equations as:

$$-\frac{d}{dt}\lambda^{(n)} = \frac{\partial H}{\partial x}\left(t, x^{(n)}, u^*(t, x^{(n)}, \Lambda^{(n-1)})\right) \tag{5.14}$$

with $\lambda^{(n)}(H) = \dfrac{\partial \phi}{\partial x}(H, x^{(n)}(H))$ reverse in time
5: Then update the co-state $\Lambda^{(n)}$

$$\Lambda^{(n)} = (1-\alpha)\Lambda^{(n-1)} + \alpha\lambda^{(n)} \tag{5.15}$$

6: Then check the stopping condition, if $\epsilon = \left\|\Lambda^{(n)} - \lambda^{(n)}\right\| < \epsilon_{max}$ then stop the simulation, else go back to step 3 and update $n := n + 1$.

The fast convergence of system depends on the choice of learning rate, but this is beyond of scope of this work.

5.4 Results

To check the model's behaviour, lets assume a test scenario. In this case suppose that follower is following the leader with a distance of $100m$ with some relative velocity. The desired distance headway is $70m$. The controller predict that leader decelerate between time 1 and 3 seconds with $-3m/s^2$ acceleration and again accelerate with $2m/s^2$ between time 5 and 7 seconds.

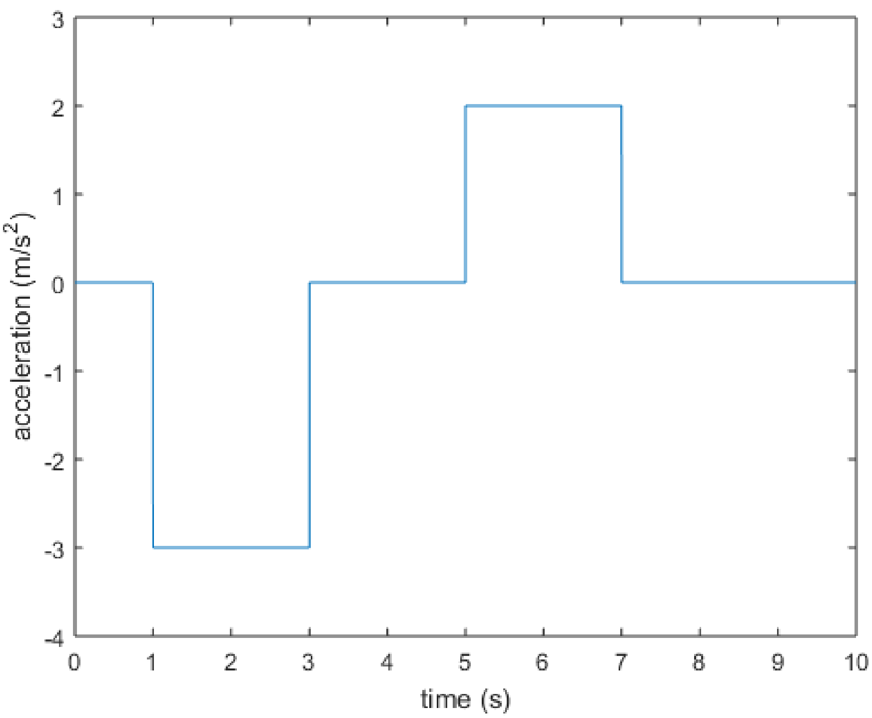

FIGURE 5.1: Leader's acceleration

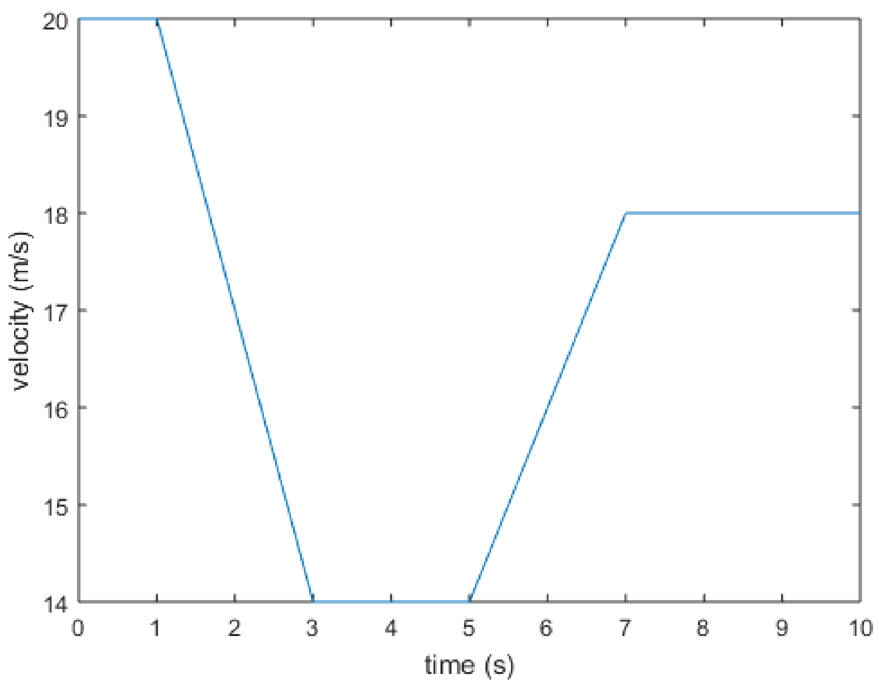

FIGURE 5.2: Leader's velocity

Figure 5.1 and 5.2 show the acceleration and velocity profile of leader respectively. The controller makes a trade-off between maintaining relative velocity at $0m/s$ and driving at desired distance headway s^*. β_1 and β_2 will decide the preference of state.

First take values of β_1 and β_2, 0.1 and 1.0 respectively and check the response. In this case, the weight of relative velocity is more as compare to relative distance. Figure 5.4 shows that the change in relative velocity has been converged to $0m/s$ but the relative distance became about $78m$ instead of $70m$(desired headway), shown in figure 5.3. Figure 5.5 and 5.6 show the manifold pressure and engine speed respectively. Figure 5.7 has co-state error which converges to 0.

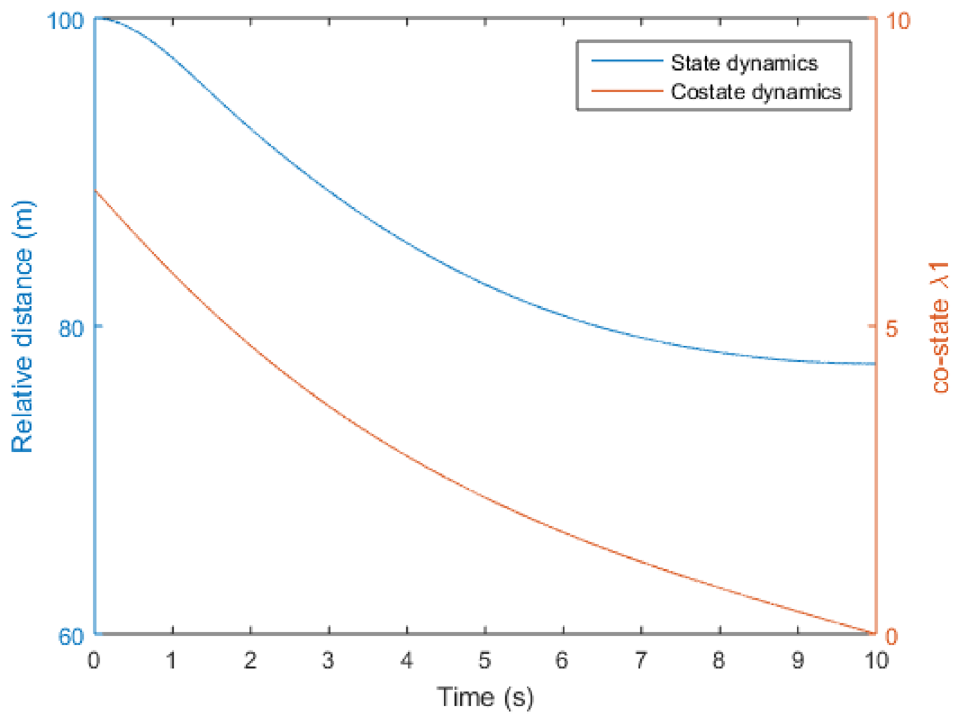

FIGURE 5.3: Relative distance and co-state λ_1

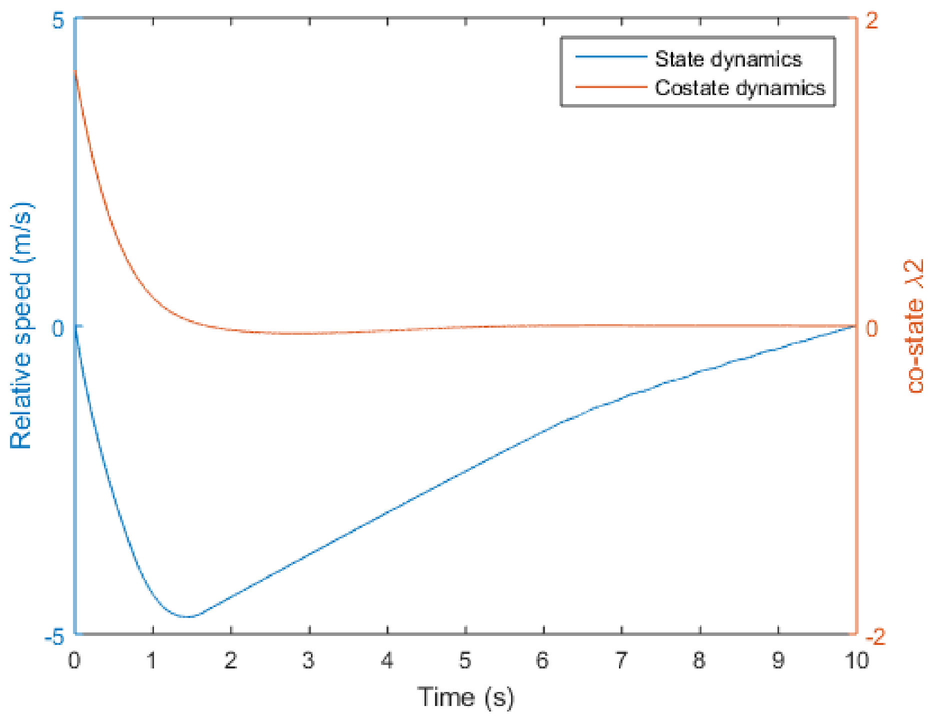

FIGURE 5.4: Relative velocity and co-state λ_2

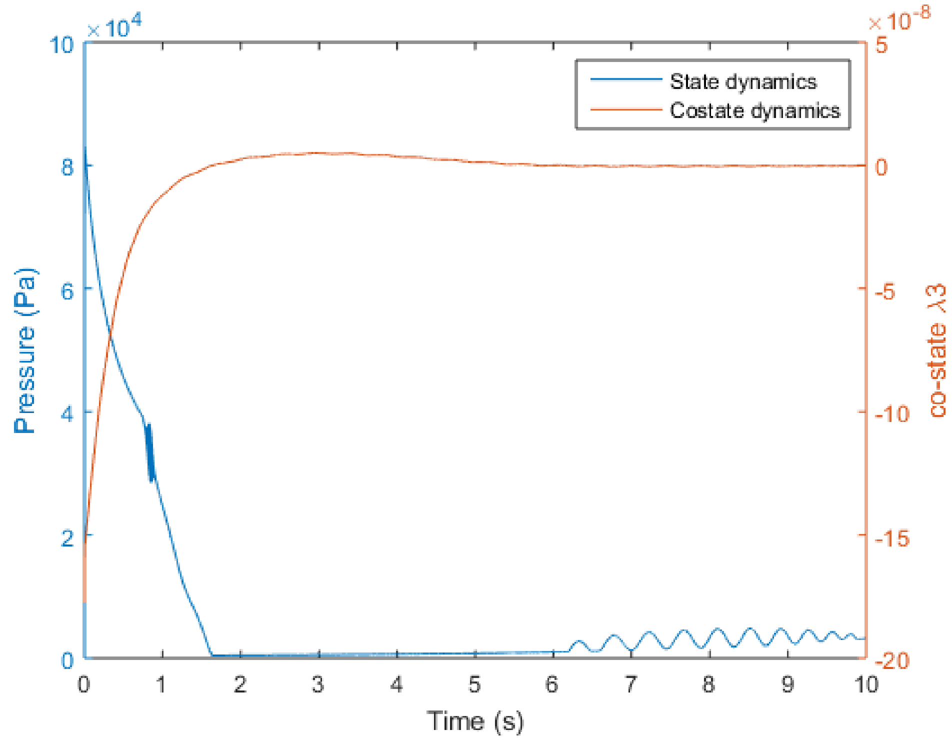

FIGURE 5.5: Manifold pressure and co-state λ_3

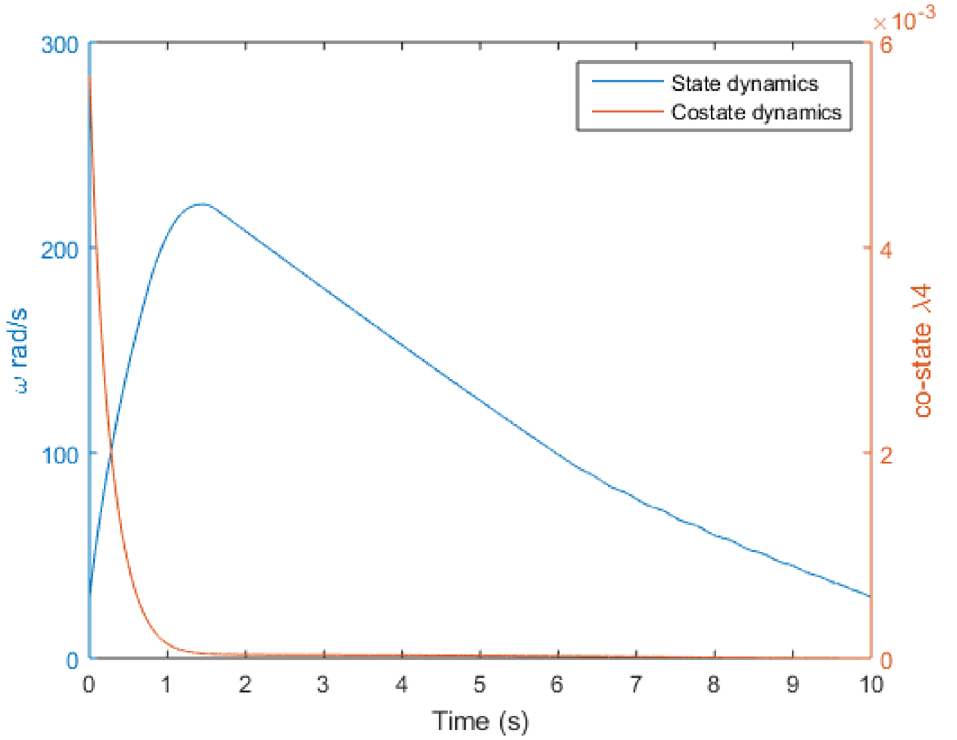

FIGURE 5.6: Engine's ω and λ_4

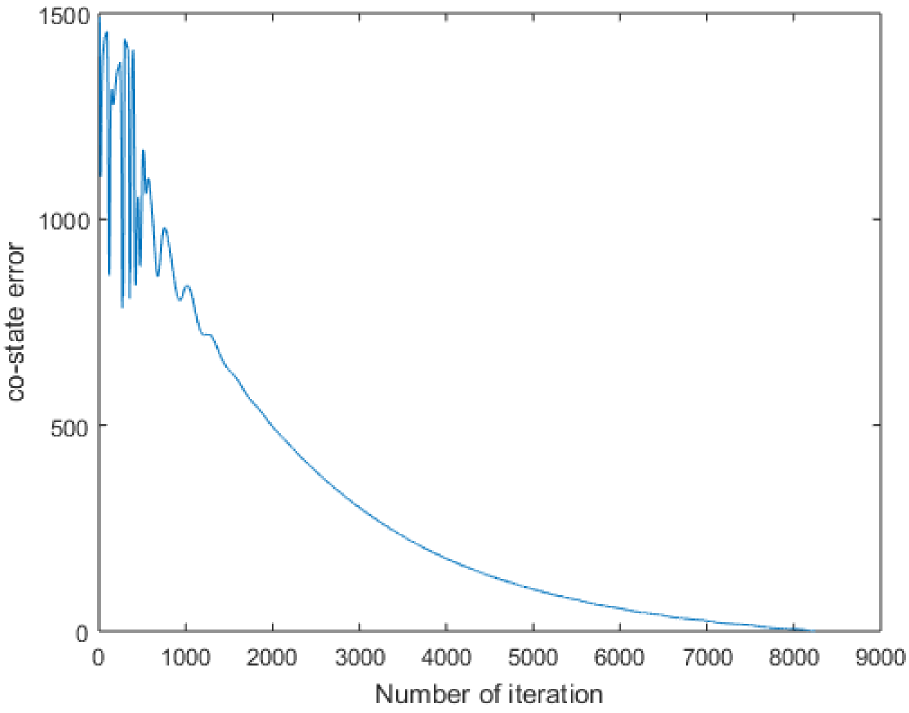

FIGURE 5.7: Co-state error

Now change the values of β_1 and β_2 as 1.0 and 0.1 respectively. The result in figure 5.8 shows that the controller is minimizing the relative distance from $100m$ to $70m$ (desire distance headway) with more priority as compare to change relative velocity which need to be $0m/s$ in figure 5.9.

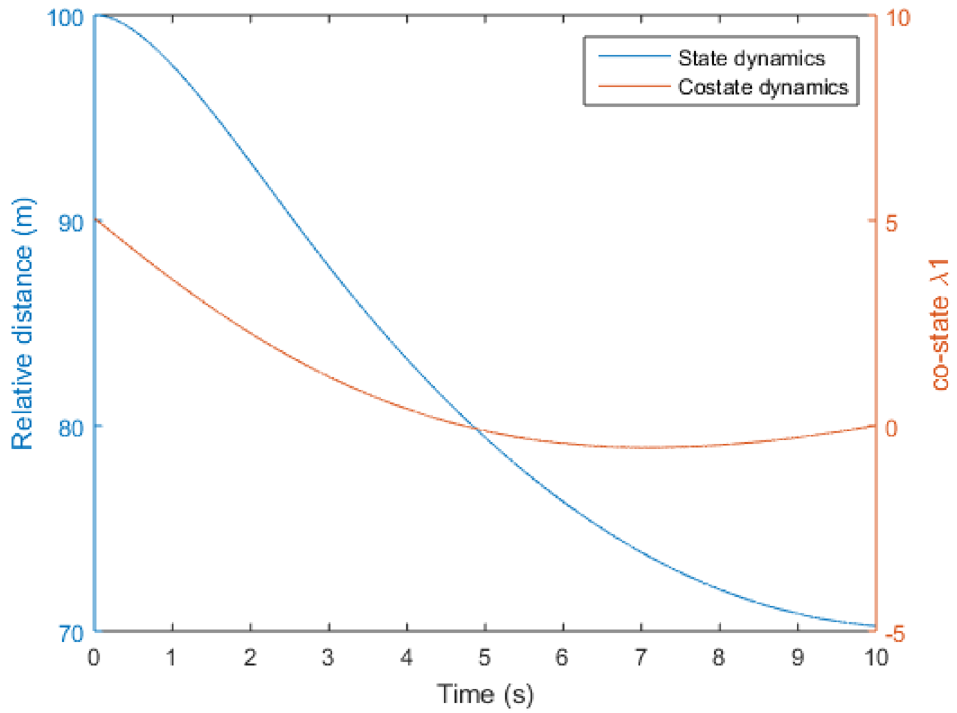

FIGURE 5.8: Relative distance and co-state λ_1

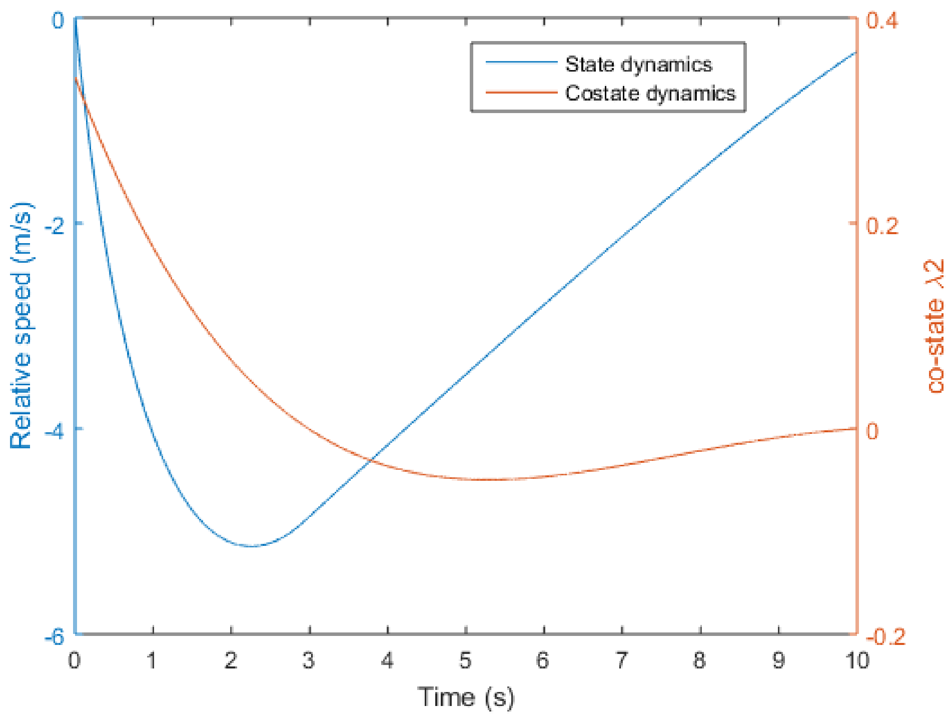

FIGURE 5.9: Relative velocity and co-state λ_2

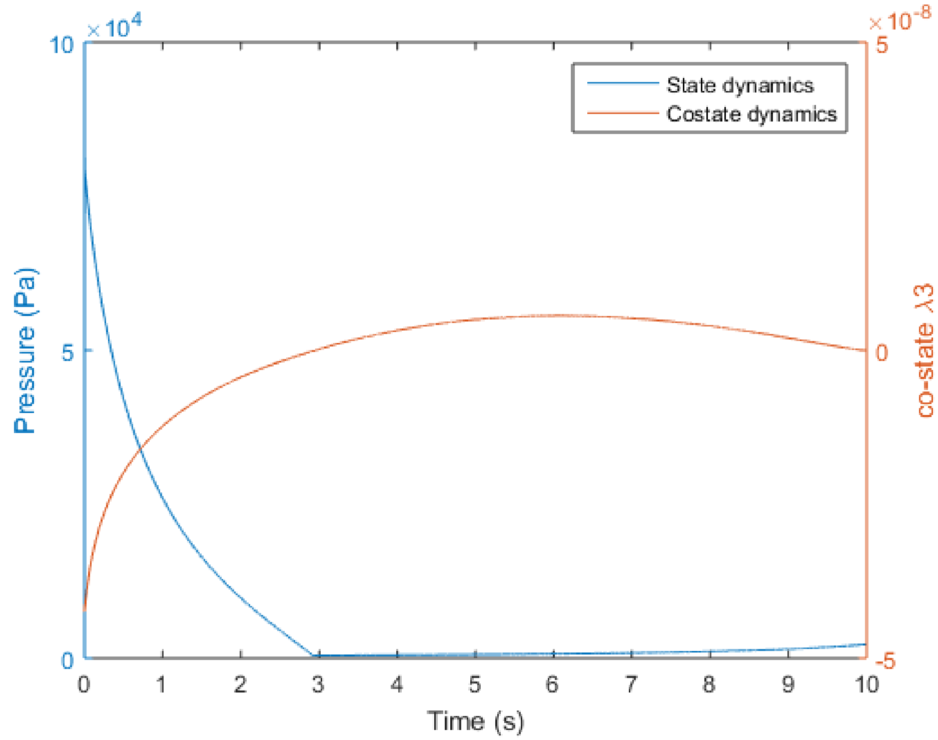

FIGURE 5.10: Manifold pressure and co-state λ_3

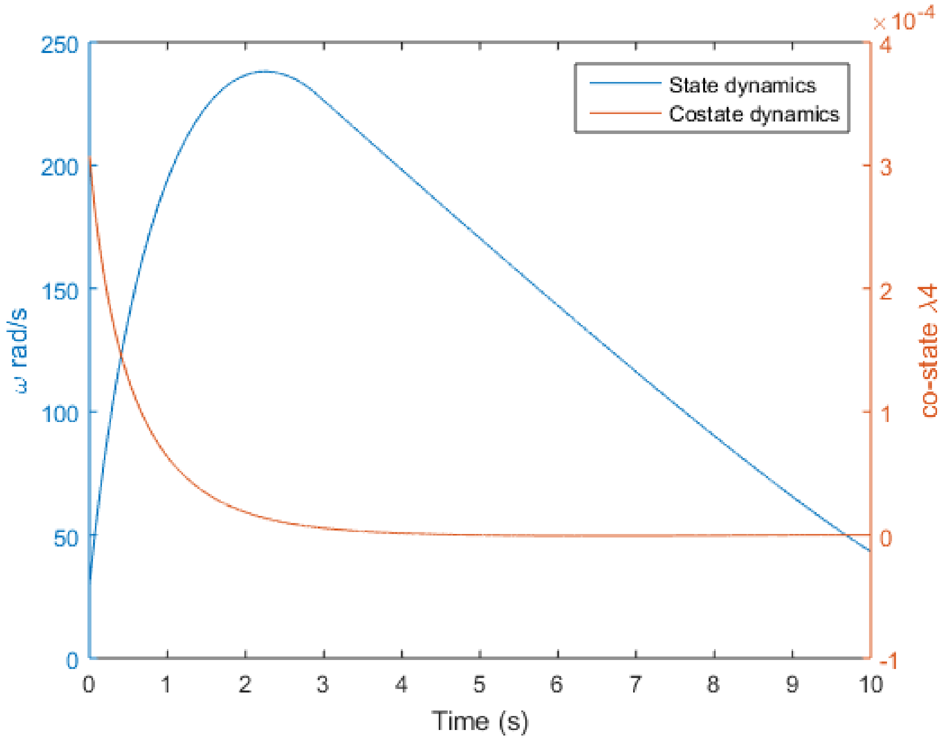

FIGURE 5.11: Engine's ω and λ_4

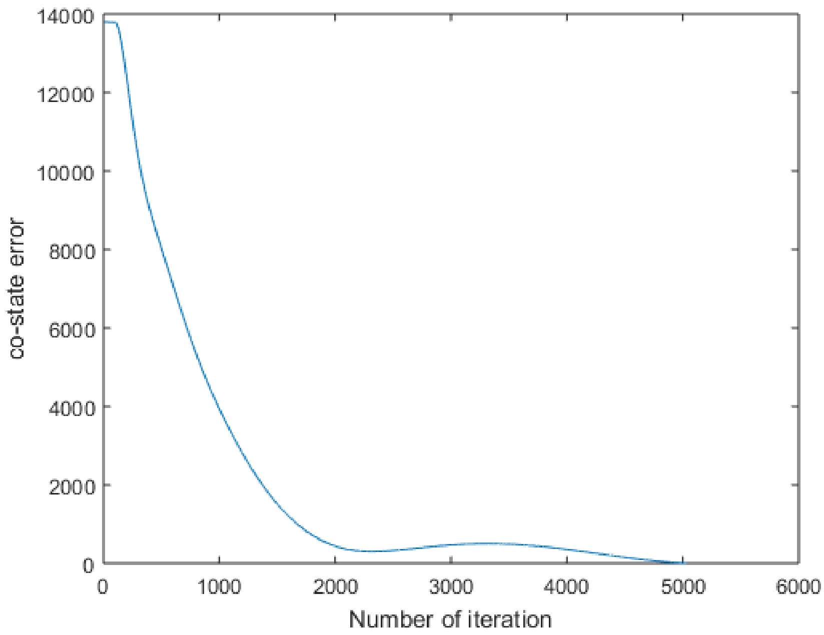

FIGURE 5.12: Co-state error

Figure 5.10 and 5.11 show the manifold pressure and engine speed respectively and figure 5.12 have co-state error which converge to 0.

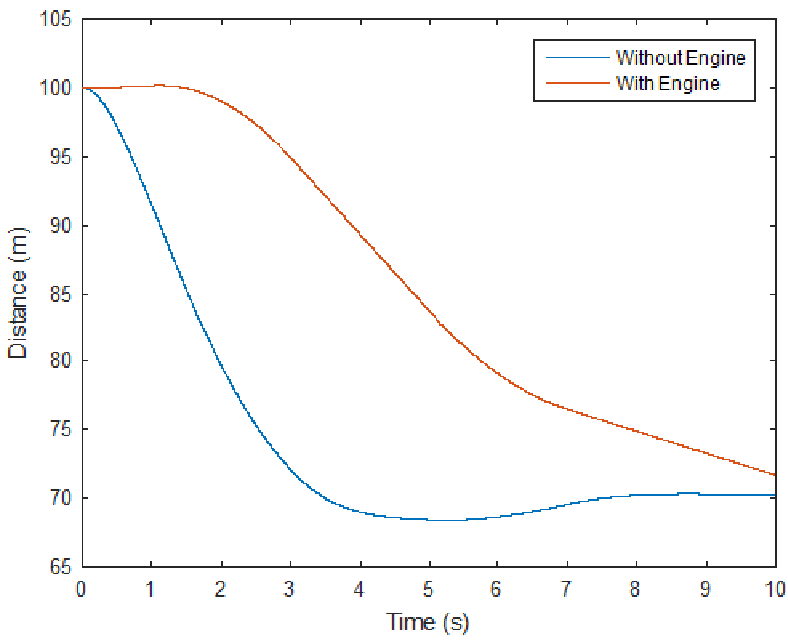

FIGURE 5.13: Relative distance between leader and followers

In Figure 5.13 the relative distance between leader and follower is shown with and without engine. The results show that the response become slower by adding engine dynamics in the system but we can realize more practical situation.

5.5 Chapter Summary

Controller designing is the main part of this thesis. In this chapter, initially, the cost formulation has been explained for maintaining a fix distance between two vehicles. To solve the system using PMP, Hamiltonian has been defined which based on the running cost, states and costates of the system. After applying PMP on the given system we have state dynamics equations with initial values and costate dynamics equation with final values. This two boundary value problem makes the solution of system difficult. To solve the system a numerical algorithm is presented in this chapter. It can be observed from the algorithm that by increasing the number of state in the system, difficulty level do not increase exponentially. At the end of this chapter result has been given. These results show that the proposed technique is promising the described problem statement.

Chapter 6

Conclusion and Future Work

This work aims, using Pontraygin's minimum principle, to design an optimal control of CACC for the purpose to maintain a fix distance headway. The system model has been obtained by joining the states of vehicle kinematic model and engine's states. A cost function has been defined based on relative distance, relative velocity and control input. The optimum control input has been calculated using PMP and result has been shown at the end. As per author's best knowledge, this is the first ever work in which the engine dynamics have been included explicitly for designing the CACC.

6.1 Conclusion

Although, all the previous work on CACC gave good result in different scenarios but no one had included the engine dynamics. The purpose of this work is to add the engine dynamics for CACC.

In this work, an optimal control is given which based on Pontraygin's minimum principle to control the vehicle speed, so that it stay on a desired headway. A numerical solution has been also given for the optimal control problem. A test scenario is given which justifies the proposed technique.

6.2 Future work

During the work several ideas have been formed for further research. An important future work lays ahead from this point onwards. Number of vehicles in the platoon can be increased, Instead of fix gear, gear shifting mechanism of engine can also be considered, Instead of fix distance headway, a variable distance headway can be taken depending on follower's velocity.

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