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Numerical study of Maxwell nanofluid over a stretching sheet along with the thermal radiation and chemical reaction effects

by

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Declaration of Authorship

I, Hifsa Khan, declare that this thesis titled, 'Numerical study of Maxwell nanofluid over a stretching sheet along with the thermal radiation and chemical reaction effects' and the work presented in it are my own. I confirm that

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- Where I have consulted the published work of others, this is always clearly attributed.
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"Education is the ability to listen to almost anything without losing your temper or your self-confidence."

Robert Frost

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Abstract

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A mathematical model is presented for analyzing the convective Maxwell fluid over a stretching porous surface in the presence of nanoparticles. The analysis of stagnation point and heat transfer of convected Maxwell fluid with slip boundary condition is investigated. To convert the governing partial differential equations (PDEs) into a system of nonlinear ordinary differential equations (ODEs) we use similarity transformations. Shooting method is used to solve the system of ODEs numerically and obtained numerical results are compared with those obtained by the built-in bvp4c function in Matlab. The numerical values obtained for the velocity, temperature and concentration profiles are presented through graphs and tables. A discussion on the effects of various physical parameters and heat transfer characteristics is also included.

Acknowledgements

All praises to Almighty **Allah**, the creator of all the creatures in the universe, who has created us in the structure of human beings as the best creature. Many thanks to Him, who created us as a Muslim and blessed us with knowledge to differentiate between right and wrong. Many many thanks to Him as he blessed us with the Holy Prophet, **Hazrat Muhammad (Sallallahu Alaihay Wa'alihi wasalam)** for Whom the whole universe is created. He (Sallallahu Alaihay Wa'alihi wasalam) brought us out of darkness and enlightened the way to heaven.

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May Almighty Allah shower His countless blessings on all those who assisted me in any way during completion of my thesis.

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Nomenclature

a	A real constant	ν	Kinematic viscosity
k_0	Relaxation time	Pr	Prandtl number
α	Thermal diffusivity	Q_0	Heat coefficient
ρ	Fluid density	M	Magnetic parameter
σ_ν	Electriacl conductivity	ρ_f	Base fluid density
c_p	Specific heat	T	Fluid temperature
τ	Ratio of heat	T_∞	Ambient fluid temperature
D_B	Brownain diffusion coefficient	D_T	Thermophoresis diffusion coefficient
C	Volume fraction	T_∞	Ambient temperature
C_∞	Ambient concentration	(u, v)	Velocity vectors
β	Maxwell parameter	(x, y, z)	Axial and normal coordinates
V	Dimensionless velocity	Pr	Prandtl number
Le	Lewis number	Nb	Brownain motion parameter
Nt	Thermophoresis parameter	η	Dimensionless normal distance
θ	Dimensionless temperature	Tr	Thermal radiation parameter
μ	Viscosity	b	Slip coefficient
$Bi1$	Temperature Biot number	$Bi2$	Concentration Biot number
ϕ	Dimensionless concentration	C_w	Fraction at wall
ψ	Stream function		

DEDICATION

I dedicate this Sincere effort to my dear **Parents** and my elegant **Teachers** who are always source of Inspiration for me and their contributions are uncounted.

Chapter 1

Introduction

The transfer of heat by the movement of fluids from one place to another is called convective heat transfer. Convective heat transfer is combination of heat diffusion and bulk fluid flow that are called conduction and advection simultaneously. In engineering problems convective heat transfer has wide applications. Large number of investigations on nanofluids (i.e mixture of fluid and nanoparticles) shows that it can improve thermal conductivity in fluids. Nanofluid is a fluid containing nanometer-sized particles called nanoparticles. These nanoparticles are made of metals, oxides, carbides etc. Nanofluids have properties that make them potentially useful in many heat transfer applications. They exhibit enhanced thermal conductivity and convective heat transfer coefficient. Choi [1] studied the enhancing thermal conductivity of fluid with nanoparticles. Eastman et al. [2] have reviewed the detailed work done on convective transport in nanofluid. The different theories of heat transfer in nanofluids are discussed by Boungiorno [3]. Kuznetsov and Nield [4] studied the convective nanofluid in vertical plate, later they extended their work for porous medium as well [5].

In [6], Makinde and Aziz studied the effect of convective boundary condition on a boundary layer flow in nanofluid. Ramesh and Gireesha [7] considered the heat source/sink of Maxwell fluid over stretching sheet with convective boundary condition in boundary layer flow in the presence of nanoparticles. In non-Newtonian fluids the behaviour of boundary layer flow seeks much interest due to many applications in manufacturing and industrial processes. The magnetohydrodynamics (MHD) flow of Maxwell fluid in thermophoresis and chemical reaction over a vertical sheet is presented by Noor [8]. Akbar et al. [9] discussed the effect of MHD and thermal radiation on Maxwell fluid over a stretching sheet. According to their observation, the elasticity number causes an enhancement in the heat transfer rate from the stretching sheet by the increase of magnetic parameter. Hayat et al. [10] worked on rotating Maxwell fluid in a porous medium and obtained analytical solution for unsteady MHD fluid.

Kumaria et al. [11] observed the MHD mixed convection stagnation-point flow of an upper convected Maxwell fluid. They concluded that with the increase in the elasticity number, reduction in the surface heat transfer, surface velocity gradient and displacement thickness was experienced. In [12–14] authors applied slip boundary conditions in an incompressible boundary layer flow. In recent years, MHD flows of viscoelastic fluids with or without heat transfer over a stretching sheets have also been addressed by some researchers [15–18].

Bhattacharyya et al. [19] analyzed the convection flow of boundary layer force and heat transfer past a porous plate with velocity and temperature slip effect. Das [20] observed the impact of partial slip, thermal radiation, chemical reaction, and temperature dependent fluid properties with constant heat flux over a premeable plate and nonuniform heat source/sink. The effects of partial slip, heat generation on the flow, thermal buoyancy and heat transfer of nanofluids are examined by Das [20]. Aminreza et al. [21] examined the effect of partial slip on flow and heat transfer of nanofluids past a stretching sheet. Zheng et al. [22] analyzed the effect of velocity slip on MHD flow and heat transfer over a porous sheet. Recently, the influence of partial slip flow and heat transfer over a stretching sheet in a nanofluid are examined by Sharma et al. [23].

Thesis contribution

In this thesis we present a review study of Cao et al. [24] and then extend the flow analysis with variable thermophysical properties. The obtained system of PDEs are transformed into a system of nonlinear coupled ODEs by using a suitable techniques. A numerical solution of the system of ODEs is obtained by using shooting method and comparison with the obtained numerical results by bvp4c code in Matlab. The numerical results are discussed for different parameters appearing in the solution.

Thesis outline

The thesis is described as follows:

In **Chapter 2**, we discuss some basic definitions of fluid, flow, heat transfer, boundary layer flow, basic governing laws, similarity transform and discussion on the shooting method. Furthermore, these concepts are used on describing the flow, heat transfer and the influence of thermophysical properties.

Chapter 3 contains a comprehensive review of Cao et al. [24]. A numerical study of incompressible, two-dimensional steady fluid flowing with convective boundary condition past a stretching porous surface has been analyzed. The constitutive equations of the flow model are solved numerically and the impact of physical parameters concerning the flow model on dimensionless temperature, velocity and concentration are presented through graphs and tables. Also a comparison of the obtained numerical results with the published results of Cao et al. [24] has been made and found that both are in excellent agreement.

In **Chapter 4**, we discuss the viscous, incompressible, time independent flow with heat transfer past a flat porous plate with thermal radiation and chemical reaction. The obtained system of ODEs are solved numerically after applying a proper similarity transformation. Graphs and tables describe the behavior of physical parameters. Numerical values of momentum, temperature and concentration have also been computed and discussed in this chapter.

Chapter 5 summarizes up the study and gives the major results obtained from the entire research and suggests recommendations for the future work.

All the references used in this study are listed in **Bibliography**.

Chapter 2

Basic definitions and governing equations

2.1 Basic Terminologies

In this chapter, some basic laws, terminologies and definitions will be explained, which will be helpful in continuing the work for the next chapters.

Definition 2.1.1. Fluid
In mathematical literature, a substance that has ability to flow and easily move and change its position is called fluid. That substance may be a liquid or gas.

Definition 2.1.2. Fluid mechanics
The area of physical sciences that deals with the behaviour of fluid at rest or in motion and the interaction of fluids with solids or other fluids at boundaries is called the fluid mechanics. It can be divided into further categories presented below.

Definition 2.1.3. Fluid statics
The branch of fluid mechanics that deals with the study of fluid and its characteristics at constant position is called the fluid statics.

Definition 2.1.4. Fluid dynamics
The branch that deals with the study of fluid in motion from one place to another is called fluid dynamics.

Definition 2.1.5. Uniform and non-uniform flows
If the magnitude and direction of velocity of the fluid at every point of flow is the same, then it is called the uniform flow. But if the velocity at every point of the flow is not the same, then the flow becomes non-uniform.

Definition 2.1.6. Steady and unsteady flows

If the fluid flow property at a specific point is independent of time, it is called steady flow, i.e.,

$$\frac{\partial P}{\partial t} = 0,$$

where P is any fluid property like pressure, density, velosity.
A flow in which fluid property is time dependent is called unsteady flow, i.e.,

$$\frac{\partial P}{\partial t} \neq 0.$$

Definition 2.1.7. Laminar and turbulent flows

The flow in which the fluid moves in a smooth path and paths never intersect each other is called laminar flow. The flow in which fluid particles do not have a definite path and the path lines also intersect each other is called turbulent flow. The velocity of fluid is not constant at every point in turbulent fluid.

Definition 2.1.8. Compressible and incompressible flows

If the density of fluid is constant in flow field, then the flow is called incompressible flow. For incompressible flow the mathematical equation is given as

$$\frac{d\rho}{dt} = 0,$$

where ρ denotes the fluid density and $\frac{D}{Dt}$ denotes material derivative.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla. \tag{2.1}$$

In Eq. (2.1) $\mathbf{V}$ is velocity of flow and ∇ denotes differential operator. In Cartesian coordinate system ∇ is given as

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}.$$

The flow in which density is not constant, is known as compressible flow.

Definition 2.1.9. Viscosity

It is the property of fluid that measures the resistance to flow. The viscosity is denoted by μ. Viscosity can be described in two different catagories.

Definition 2.1.10. Dynamic viscosity

The measurement of internal resistance of fluid is called dynamic viscosity. Mathematically, it is defined as the ratio of shear stress to the rate of shear strain and it is denoted by μ.

$$\text{Viscosity}(\mu) = \frac{\text{Shear stress}}{\text{Rate of shear strain}}.$$

In the above expression the coefficient of viscosity μ is also known as absolute viscosity or dynamic viscosity or simply viscosity. The SI unit of viscosity is kg/ms or Pascal-second [Pa.s].

Definition 2.1.11. Kinematic viscosity
It is the ratio of dynamic viscosity μ to the density of the fluid and it is represented by ν, Mathematically

$$\nu = \frac{\mu}{\rho}.$$

Its unit in SI system is m^2/s. ρ is the density of fluid.

Definition 2.1.12. Newtonian and non-newtonian fluids
The fluid in which the stress is linearly related to the deformation rate, is called Newtonian fluid. Newtonian fluid behaviour is written as

$$\tau = \mu \frac{du}{dy}.$$

In the above equation, τ denotes stress tensor, viscosity is μ and deformation rate is $\frac{du}{dy}$. Fluids in which the shear stress is not linearly related to deformation rate are known as non-newtonian fluids.

Definition 2.1.13. Generalized continuity equation
Law of conservation of mass states that mass can neither be created nor destroyed inside a control volume is the base of continuity equation. The mass will not be changed inside the fixed control system. If we examine a control volume system, then the continuity equation can be written as

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{V}) = 0. \tag{2.2}$$

If the density is constant, Eq. (2.2) becomes

$$\nabla \cdot \mathbf{V} = 0.$$

Definition 2.1.14. Generalized momentum equation
For fluid particles, the equation of generalized linear momentum is observed from the Newton's second law of motion. It is stats as: "The net force $\mathbf{F}$ is equal to the rate of change of linear momentum with time." Newton's second law can be written as

$$m \frac{D\mathbf{V}}{Dt} = \mathbf{F}.$$

The differential equation for flow of the fluid is represented as

$$\rho \frac{D\mathbf{V}}{Dt} = \nabla.\tau + \rho \mathbf{b},$$

where $\rho\mathbf{b}$ denotes net body force, $\nabla.\tau$ denotes surface forces and τ denotes Cauchy stress tensor.

Definition 2.1.15. Heat transfer
Energy transfer due to temperature difference is called heat transfer. Heat transfer can occur through conduction, convection or radiation.

Definition 2.1.16. Conduction
Conduction is a process in which the heat is transferred between those objects that are in direct contact with each other.

Definition 2.1.17. Convection
Transfer of heat through fluids (gases or liquids) from hot places to cold places is called convection. Convective heat transfer depends on the nature of the flow. Water boiling in pan is good example of convection. There are three forms of convection : forced convection, natural convection or free convection and mixed convection.

Definition 2.1.18. Forced convection
Convection which is produced by heat transport process by an external source is called forced convection. In other words a heat transfer technique in which fluid motion is developed by an independent source like a fan and pump etc is called forced convection.

Definition 2.1.19. Natural convection
The fluid motion is not generated by any external source in heat transport process, but the density differences in the fluid occurring due to temperature gradient. Natural convection is also called free convection.

Definition 2.1.20. Mixed convection
It occurs by combined effect of forced and natural convection to transfer heat. In other words, when both natural and forced convection processes simultaneously contribute to cause heat transfer, mixed convection appears.

Definition 2.1.21. Radiation
Radiation is a transfer of energy due to discharge of electromagnetic waves from a surface volume. It doesn't need any medium to transfer heat.

Definition 2.1.22. Thermal conductivity
Thermal conductivity (κ) is the property of a material related to its capability to conduct heat. Mathematically,

$$\kappa = \frac{q\nabla l}{S\nabla T},$$

where q is the heat passing through a surface area S, causing a temperature difference ∇T over a distance of ∇l. Here l, S and ∇T are all assumed to have unit measurement. The unit of thermal conductivity in SI unit is $\frac{W}{m.\kappa}$ and its dimension is $[MLT^{-3}\theta^{-1}]$.

Definition 2.1.23. Thermal diffusivity
Thermal diffusivity is material property for unsteady heat conduction. Mathematically, it can be expressed as,

$$\alpha = \frac{\kappa}{\rho C_p},$$

where κ, ρ and C_p represent the thermal conductivity of material, the density and the specific heat capacity respectively. The unit and dimension of thermal diffusivity in SI system are m^2s^{-1} and $[LT^{-1}]$ respectively.

Definition 2.1.24. Prandtl number Pr
It is the ratio between the momentum diffusivity (ν) and the thermal diffusivity (α). It is the dimensionless number. Mathematically it can be written as,

$$Pr = \frac{\nu}{\alpha} \quad \Longrightarrow \quad \frac{\mu/\rho}{k/c_p} \quad \Longrightarrow \quad \frac{\mu c_p}{k},$$

where μ represents the dynamic viscosity, C_p the specific heat and κ stands for thermal conductivity. The relative thickness of thermal and momentum boundary layer are controlled by Prandtl number. For small Pr, heat is distributed rapidly corresponded to the momentum.

Definition 2.1.25. Lewis number Le
It is the ratio of thermal diffusivity to the mass diffusivity. Mathematically,

$$Le = \frac{\alpha}{D_B},$$

where α is the thermal diffusivity and D_B the Brownian mass diffusivity.

Definition 2.1.26. Reynolds number R_e
It is used to predict flow pattern weather the flow is turbulent or laminar. It is the ratio of inertial force to viscous force. The formulae is given by

$$R_e = \frac{\rho V L}{\mu} \tag{2.3}$$

where ρ is density, V is velocity, L is length and μ is viscosity of fluid.

Definition 2.1.27. Schmidt number (Sc)
Schmidt number (Sc) is defined as the ratio of momentum diffusivity (viscosity) and mass diffusivity. It can be written as

$$Sc = \frac{\nu}{D_B} \tag{2.4}$$

Definition 2.1.28. Sherwood number (Sh)

It is used in mass transfer ratio. It is also called nusselt number.It is define as

$$Sh = \frac{K}{D/L} \tag{2.5}$$

Definition 2.1.29. Boundary layer flow

A layer of reduced velocity in fluid is called boundary layer (including the matter like air and water) and this in return is exactly adjacent to solid surface which is just following the fluid. The basic idea of boundary layer in motion of a fluid over a surface was first introduced by Ludwig Prandtl (1874-1953). Heat transfer and skin friction are due to the basic ideas and knowledge introduced by him in twentieth century. The reason why we have the velocity zero's exactly adjacent to the layer is that the viscous effect and the layer of fluid which makes contact with the surface becomes slowly adhered to the surface resulting in a condition of no-slip. The phenomenon of shearing takes place due to the fact that the layers of fluid are moving. The ratio of the two important forces determined by the Reynolds number play a vital role in determination of the thickness of the boundary layer. There are two types of boundary layers:

- Hydrodynamic boundary layer
- Thermal boundary layer

Definition 2.1.30. Hydrodynamic boundary layer

A region of a fluid flow where the transition from zero velocity at the solid surface to the free stream velocity at some extent far from surface in the direction normal to the flow takes place in a very thin layer, is known as hydrodynamic boundary layer.

Definition 2.1.31. Thermal boundary layer

The heat transfer exchange surface and the free stream have a liquid or a gaseous agent for heat transfer. From wall to free stream we come across the change of temperature of heat transfer agent. Fluid velocity and profile change is similar. It increases from wall to main stream. The surface temperature is assumed to be equal to the temperature of the fluid layer closed to the wall inside the boundary and this temperature is equal to the temperature of the bulk at some point in the fluid.

Definition 2.1.32. Similarity transformation

Similarity Transformation is a mathematical technique that is applicable in some cases by which the PDEs of a problem are converted into the ODEs. This technique obviously reduces the number of independent variables of the problem to one. It can be viewed as a rule for combining the two independent variables into a new variable. [24]

Definition 2.1.33. Shooting method
Shooting Method is a numerical technique used to solve the BVPs for ODEs. It is an iterative techniquew that transform the original BVPs to the related IVPs with its initial conditions. The formulated problem requires the IVP solution with arbitrary chosen initial conditions to approximate the boundary conditions at the end point. If the boundary conditions are not fulfilled to the required accuracy, with the new set of initial conditions the method is repeated again until the required accuracy is achieved or to the limit of the iteration is reached then the resulting IVP numerically solved by any appropriate method for solving the linear ordinary differential equation.The advantages of shooting methods are:

- Easy and simple.
- Fast and efficient.
- Requires less storage.

Chapter 3

Convectection of Maxwell fluid over stretching porous surface in the presence of nanoparticles

3.1 Introduction

In this chapter, we examine the influence of heat source/sink of Maxwell fluid in the steady boundary layer flow in the presence of nanoparticles with the convective boundary condition. The study of convective heat transfer has great importance in engineering problems. Furthermore, nanoparticles can improve thermal conductivity, when added to the base fluid. We describe the flow equations which are then transformed into a system of coupled non-linear ODEs by implementing a proper similarity transformation. These modeled ODEs are solved numerically by using shooting method. Finally the numerical results are discussed at the end of the chapter for various physical parameters affecting the flow and heat transfer and found to be in excellent agreement with those obtained by the Matlab bvp4c code. Graphs and tables are also discussed to show the significance of the physical parameters. This chapter provides the detailed review of Cao et al. [24].

3.2 Mathematical modeling

Consider the incompressible, two-dimensional and steady flow of a fluid with heat transfer past a porous surface in nanoparticles. The geometry of flow model is given in 3.1

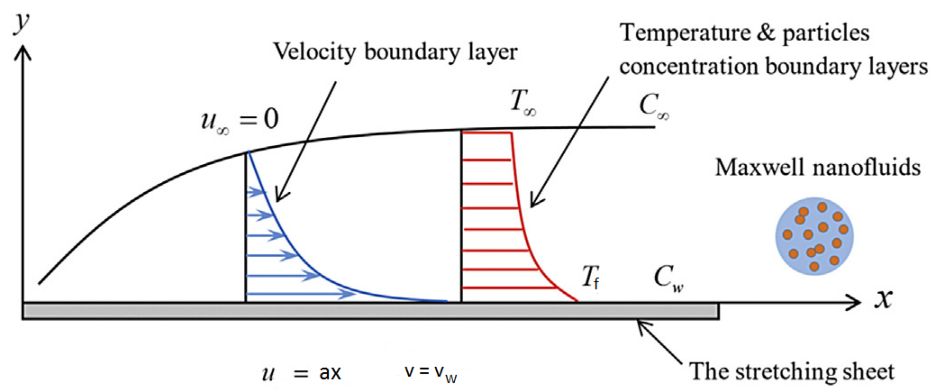

Figure 3.1: Geometry for the flow under consideration.

The governing equations obeying the boundary layer theory can be written as,

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{3.1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+k_0(\bar{u}^2\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\bar{v}^2\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+2\bar{u}\bar{v}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}), \tag{3.2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\alpha\frac{\partial^2 T}{\partial \bar{y}^2}+\frac{Q_0}{\rho_f c_p}(T-T_\infty)+\tau(D_B(\frac{\partial C}{\partial \bar{y}}\frac{\partial T}{\partial \bar{y}})+\frac{D_T}{T_\infty}(\frac{\partial T}{\partial \bar{y}})^2), \tag{3.3}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}}+\bar{v}\frac{\partial C}{\partial \bar{y}}=D_B(\frac{\partial^2 C}{\partial \bar{y}^2})+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial \bar{y}^2}. \tag{3.4}$$

In Eqs. (3.1) - (3.4), $\bar{u}$ and $\bar{v}$ are the velocities in the $\bar{x}-$ and $\bar{y}-$ directions respectively, α denotes thermal diffusivity, base fluid density is ρ_f, ν denotes fluid kinematic viscosity, T denotes the fluid temperature, ambient temperature is T_∞, k_0 denotes relaxation time, Q_0 denotes heat generation, Brownian diffusion coefficient D_B, D_T denotes the thermophoretic diffusion coefficient, c_p is the specific heat of the constant pressure, τ denotes the effective heat capacity and nanoparticle volume fraction is C.

The associated boundary conditions for the above system of equations are,

$$\bar{u}=a\bar{x}, \quad \bar{v}=v_w, \quad -k\frac{\partial T}{\partial \bar{y}}=h_f(T_f-T), \quad C=C_w, \quad \text{at} \quad y=0, \tag{3.5}$$

$$\bar{u}\to 0, \quad T\to T_\infty, \quad C\to C_\infty \quad \text{as} \quad y\to\infty. \tag{3.6}$$

In Eqs. (3.5) and (3.6), $a>0$ is a constant, C_w denotes the wall fraction of nanoparticles, v_w shows the velocity at wall, k is the effective thermal conductivity of nanofluid and the ambient nanoparticle volume fraction is C_∞.

3.3 Similarity transformation

In this section, we convert the system of Eqs. (3.1) - (3.4) along with the boundary conditions (3.5) - (3.6) into a dimensionless form. To find out the solution of our model, we use the following similarity transformation.

$$x = \frac{\bar{x}}{\sqrt{\frac{\nu}{a}}}, \quad y = \frac{\bar{y}}{\sqrt{\frac{\nu}{a}}}, \quad u = \frac{\bar{u}}{\sqrt{a\nu}}. \quad v = \frac{\bar{v}}{\sqrt{a\nu}}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi = \frac{C - C_w}{C_w - C_\infty}.$$

The stream function ψ defined by $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ leads to the following dimensionless form of (3.2) - (3.4):

$$\psi_x \psi_{xy} - \psi_x \psi_{yy} - \psi_{yyy} - \beta((\psi_y)^2 \psi_{xxy} + (\psi_x)^2 \psi_{yyy} - 2\psi_x \psi_y \psi_{xyy}) = 0, \tag{3.7}$$

$$\psi_y \theta_x - \psi_x \theta_y - \frac{1}{Pr}\theta_{yy} - S\theta - Nb\phi_y \theta_y - Nt(\theta_y)^2 = 0, \tag{3.8}$$

$$LePr(\psi_y \phi_x - \psi_x \phi_y) - \phi_{yy} - \frac{Nt}{Nb}\theta_{yy} = 0, \tag{3.9}$$

where $\beta = ak_0$ is the Maxwell parameter, $Nb = \frac{\tau D_B(C_w - C_\infty)}{\nu}$ is the parameter of Brownian motion, $Nt = \frac{\tau D_T(T_f - T_\infty)}{\nu T_\infty}$ is the thermophoresis parameter, $S = \frac{Q_0}{a\rho_f c_p}$ is the heat source ($S > 0$) or sink ($S < 0$) parameter, $Le = \frac{\alpha}{D_B}$ is Lewis number and $Pr = \frac{\nu}{\alpha}$ is Prandtl number.

After applying the stream function, the corresponding boundary conditions for the velocity components expressed in Eqs. (3.5) and (3.6) would be,

$$\psi_y = x, \quad \psi_x = M, \quad \theta_y = Bi(\theta - 1), \quad \phi = 1, \quad \text{at} \quad y = 0 \tag{3.10}$$

$$\psi_y \to 0, \quad \theta \to 0, \quad \phi \to 0, \quad \text{as} \quad y \to \infty. \tag{3.11}$$

where $M = -\frac{v_w}{\sqrt{a\nu}}$ is the premeability parameter and $Bi = \frac{h_f}{k}\sqrt{\frac{\nu}{a}}$ is the Biot number.

The corresponding similarity functions and variables are

$$\eta = y, \ \psi = xG(\eta), \ \theta = \theta(\eta), \ \phi = \phi(\eta). \tag{3.12}$$

The differential Eqs. (3.7) - (3.9) with the associated boundary conditions (3.10) - (3.11) take the following form after applying the similarity transformation (3.12).

$$(1 + \beta G^2)G''' + GG'' + 2\beta GG'G'' - (G')^2 = 0, \tag{3.13}$$

$$\frac{1}{Pr}\theta'' + G\theta' + S\theta + Nb\theta'\phi' + Nt(\theta')^2 = 0, \tag{3.14}$$

$$\phi'' + LePrG\phi' + \frac{Nt}{Nb}\theta'' = 0. \tag{3.15}$$

The boundary conditions become

$$G(\eta) = M, \quad G'(\eta) = 1, \quad \phi(\eta) = 1, \quad \theta'(\eta) = Bi(\theta(\eta) - 1), \quad \text{at} \qquad \eta = 0, \tag{3.16}$$

$$G'(\eta) \to 0, \quad \theta(\eta) \to 0, \quad \phi(\eta) \to 0, \quad \text{as} \quad \eta = \infty. \tag{3.17}$$

3.4 Solution methodology

The analytic solution of the system of equations with corresponding boundary conditions (3.13) - (3.17) cannot be found because they are highly non linear and coupled. So we use a numerical technique, i.e., shooting technique with fourth order Runge-Kutta method [25]. In order to solve the system of ODEs (3.13) - (3.15) with boundary conditions (3.16) - (3.17) using shooting method, we have to first convert these system of equations into a system of first order differential equations. For this purpose, let

$$\begin{aligned} &G = y_1, \quad G' = y_2, \quad G'' = y_3, \quad G''' = y_3', \\ &\theta = y_4, \quad \theta' = y_5 \quad , \quad \theta'' = y_5', \\ &\phi = y_6 \quad , \quad \phi' = y_7 \quad , \quad \phi'' = y_7'. \end{aligned} \tag{3.18}$$

The coupled nonlinear momentum, temperature and concentration equations are then converted into the following system of seven first order differential equations and the corresponding boundary conditions are transformed into the following form:

$$y_1' = y_2, \tag{3.19}$$

$$y_2' = y_3, \tag{3.20}$$

$$y_3' = \frac{1}{1 + \beta y_1^2}(-y_1 y_3 - 2\beta y_1 y_2 y_3 + (y_2)^2), \tag{3.21}$$

$$y_4' = y_5, \tag{3.22}$$

$$y_5' = Pr(-y_1 y_5 - S y_4 - Nb y_5 y_7 - Nt(y_5)^2), \tag{3.23}$$

$$y_6' = y_7, \tag{3.24}$$

$$y_7' = -LePr y_1 y_6 - \frac{Nt}{Nb} y_5', \tag{3.25}$$

$$y_1(0) = M, \; y_2(0) = 1, \; y_5(0) = Bi(y_4(0) - 1), \; y_6(0) = 1. \tag{3.26}$$

The shooting method requires the initial guess for $y_3(\eta)$, $y_4(\eta)$ and $y_7(\eta)$ at $\eta = 0$, and through Newton's method we vary each guess until we obtain an approximate solution for our problem. To check accuracy of the obtained numerical results by shooting method we compare them by the numerical results acquired by Matlab bvp4c solver and found them in excellent agreement.

3.5 Results and discussion

The objective of this section is to analyze the numerical results displayed in the form of tables and graphs. The computations are carried out for various values of the effects of Maxwell parameter β, Biot number Bi, Prandtl number Pr, Brownian motion parameter Nb, Thermophoresis parameter Nt, parameter M, heat source parameter S and hence the influence of these parameters on velocity, temperature and concentration profile are discussed.

The comparison of present results with those obtained by Makinde and Aziz [6] corresponding to -$\theta(0)$ and -$\phi(0)$ is presented in Table 3.1. It shows that increases in N_T cause decrease in both $-\theta'(0)$ and $-\phi'(0)$. The numerical value of -$\theta(0)$ and -$\phi(0)$ for various value of Pr, S and M when other parameters are kept fixed are present in Table 3.2 which indicate that increase in Prandtl number cause increase in both $-\theta'(0)$ and $-\phi'(0)$.

	Present		**Makinde and Aziz**[6]	
$\boldsymbol{Nt}$	$-\theta'(0)$	$-\phi'(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.092907	2.277399	0.0929	2.2774
0.2	0.092733	2.248939	0.0927	2.2490
0.3	0.092545	2.222793	0.0925	2.2228
0.4	0.092344	2.199173	0.0923	2.1992
0.5	0.092126	2.178327	0.0921	2.1783

TABLE 3.1: Comparison of -$\theta'(0)$ and -$\phi'(0)$ for $Pr = 10$, $Le = 1$, $Nb = 0.1$ and $\beta = M = S == 0$

	$S = 0.5$, $M = 1$		$S = -0.5$, $M = 1$		$S = M = 0$	
$\boldsymbol{Pr}$	$-\theta'(0)$	$-\phi'(0)$	$-\theta'(0)$	$-\phi'(0)$	$-\theta'(0)$	$-\phi'(0)$
2	0.094950	2.409880	0.096213	2.406364	0.089191	0.867379
4	0.097118	4.544080	0.097616	4.542003	0.091777	1.340429
6	0.097815	6.616794	0.098127	6.615211	0.092622	1.704483
8	0.098143	8.663921	0.098378	8.662570	0.092914	2.012471

TABLE 3.2: Values of -$\theta'(0)$ and -$\phi'(0)$ for $Nb = Nt = \beta = Bi = 0.1$ and $Le = 1$

Figure 3.2 shows that when Maxwell parameter increases, boundary layer thickness also increases. Figure 3.3 gives the effect of Biot number in the presence of heat source/sink on temperature and Figure 3.4 shows the effect of Biot number on concentration. It is found that temperature and concentration are increasing functions of Biot number. It is also be observed that heat source and heat sink has minor effect on the concentration $\phi(0)$. Figure 3.5 shows the impact of Le on temperature $\theta(0)$ and Figure 3.6 shows the effect of Le on concentration $\phi(0)$. Temperature and concentration are decreasing function of Lewis number. Figure 3.7 shows the effect of Prandtl number on temperature and Figure 3.8 shows the effect of Prandtl number on concentration. Both are decreasing function of Pr. Heat source is higher than heat sink in temperature for fixed Prandtl number. Figures 3.9 shows the impact of Brownian motion in temperature and Figure 3.10 shows the effect of Nb on concentration. The temperature increases with the increase of Brownian motion. The influence near the wall is high. The Brownian motion parameter is the decreasing function of concentration. Figure 3.11 and 3.12 shows the impact of thermophoresis parameter on the temperature and concentration simultaneously. Both temperature and concentration are increasing functions of Nt. Figure 3.13 and 3.14 shows the effect of suction velocity in both temperature and concentration respectively. Figure 3.15 shows the effect of heat source on temperature.

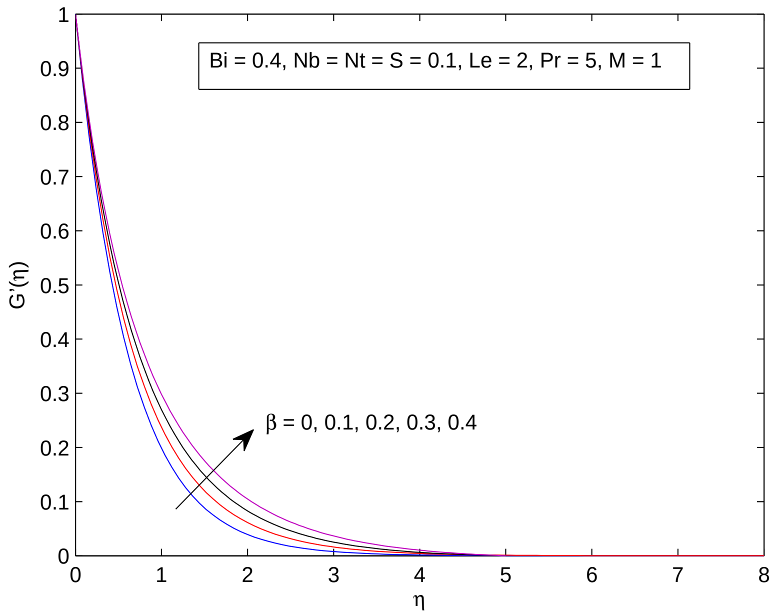

FIGURE 3.2: Impact of Maxwell parameter β on velocity $G'(\eta)$.

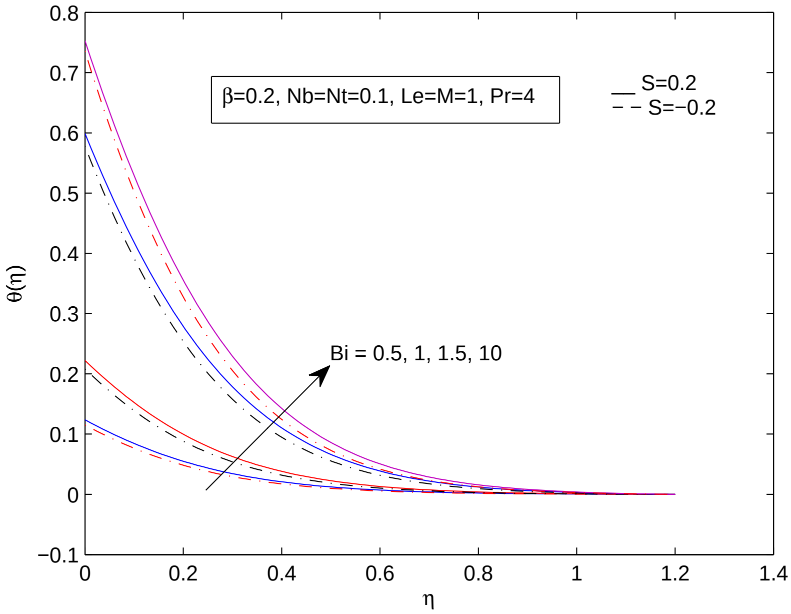

FIGURE 3.3: Effect of Biot number Bi 0n temperature $\theta(\eta)$

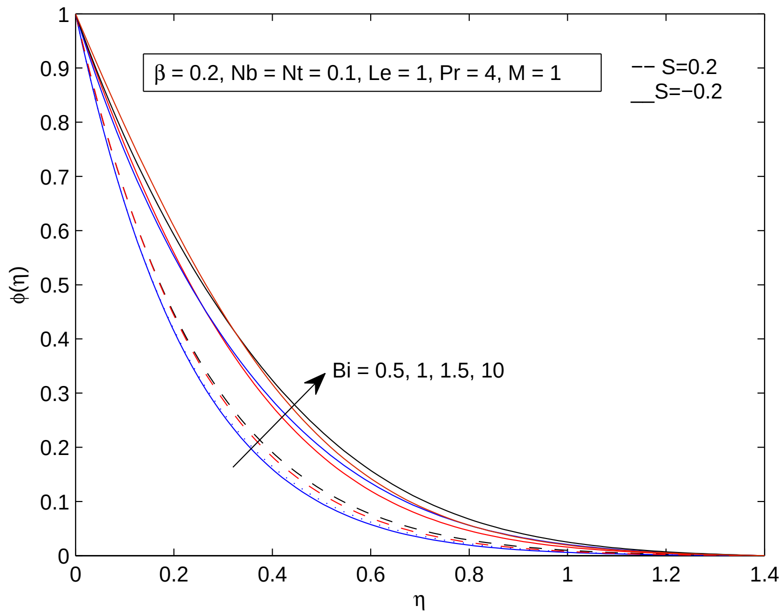

FIGURE 3.4: Effects of Biot number Bi on concentration $\phi(\eta)$

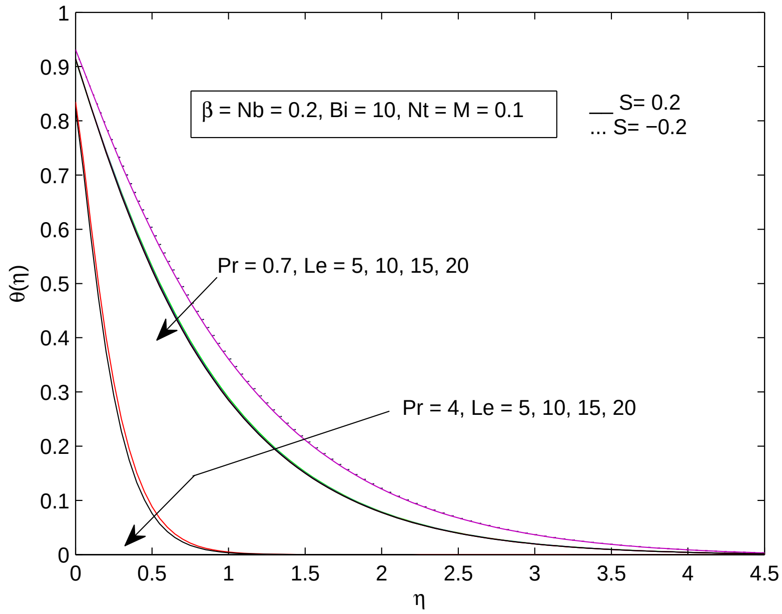

FIGURE 3.5: Effects of Lewis number Le on temperature $\theta(\eta)$

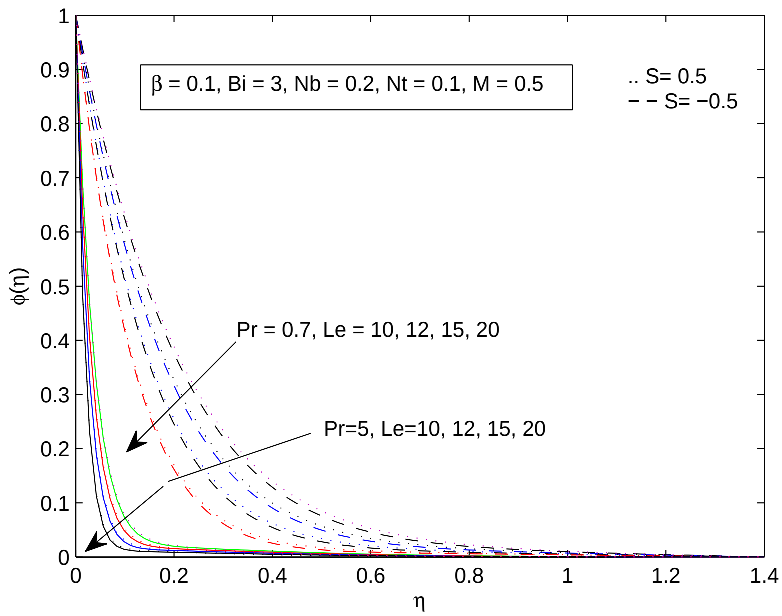

FIGURE 3.6: Effects of Lewis number Le on concentration $\phi(\eta)$

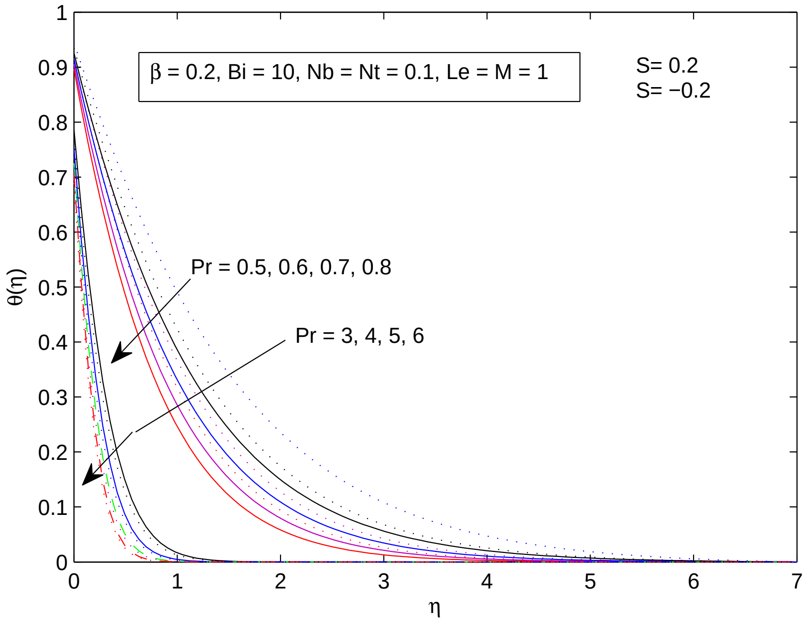

FIGURE 3.7: Effects of Prandtl number Pr on temperature $\theta(\eta)$

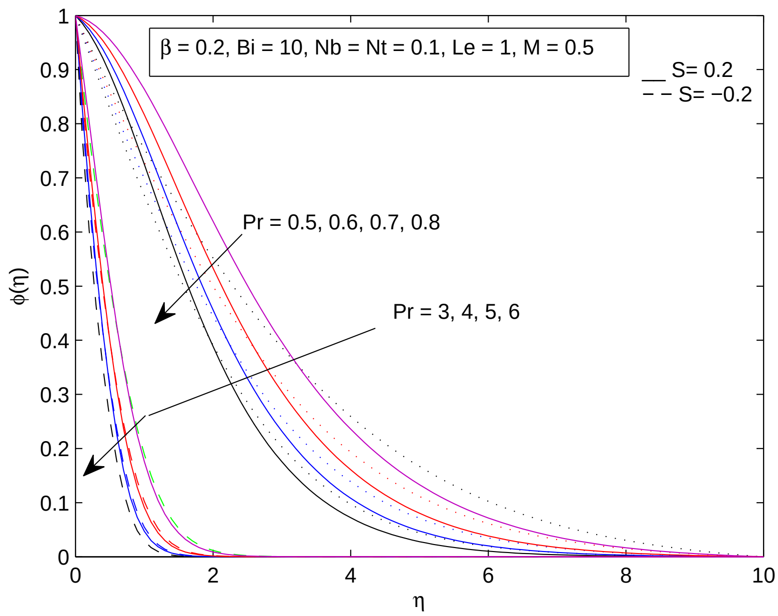

FIGURE 3.8: Effects of Prandtl number Pr on concentration $\phi(\eta)$

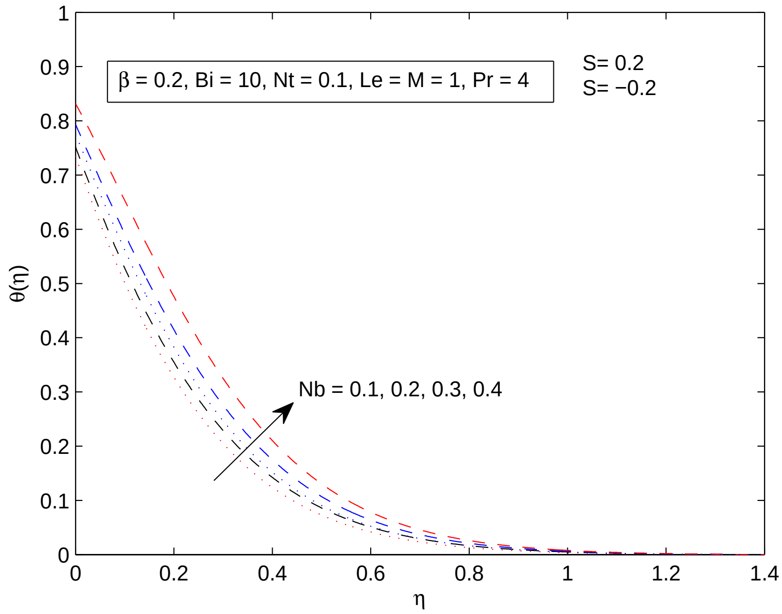

FIGURE 3.9: Effects of Brownian motion parameter Nb on temperature $\theta(\eta)$

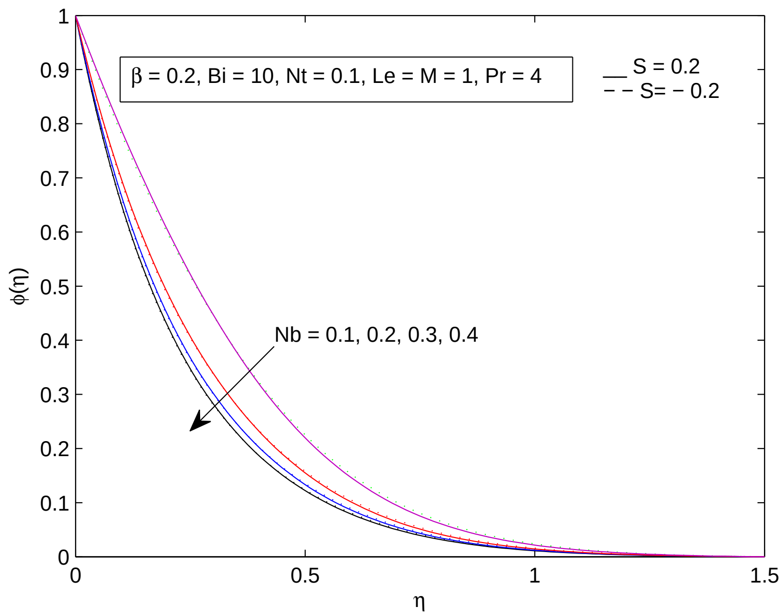

FIGURE 3.10: Effects of Brownian motion parameter Nb on concentration $\phi(\eta)$

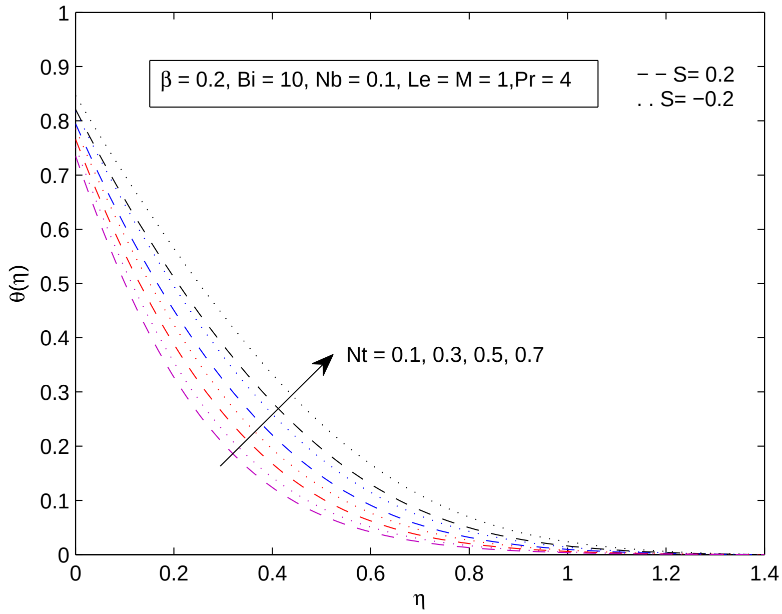

FIGURE 3.11: Impact of Nt on temperature $\theta(\eta)$.

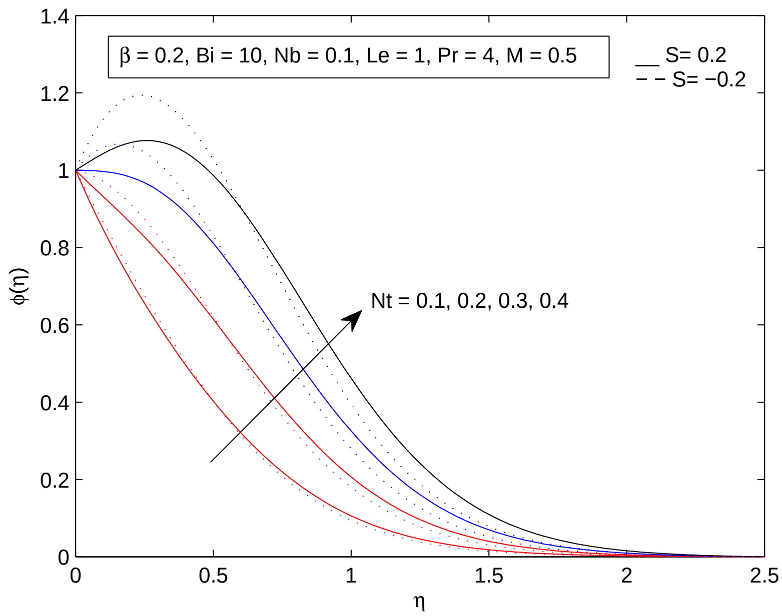

FIGURE 3.12: Impact of Nt on concentration $\phi(\eta)$

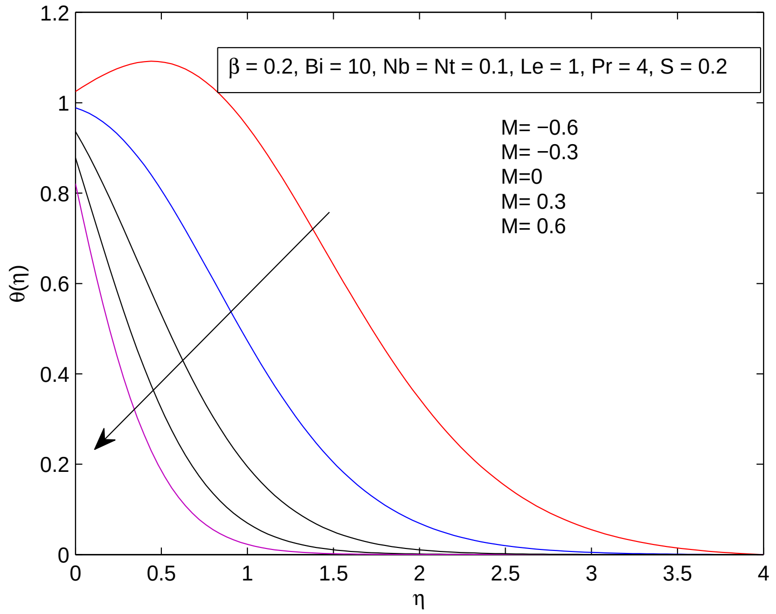

FIGURE 3.13: Impact of parameter M on temperature $\theta(\eta)$

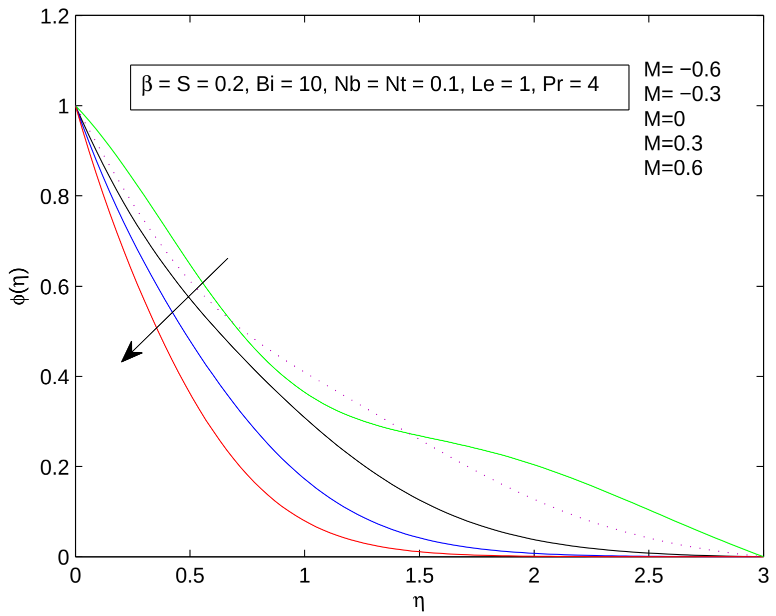

FIGURE 3.14: Impact of parameter M on concentration $\phi(\eta)$

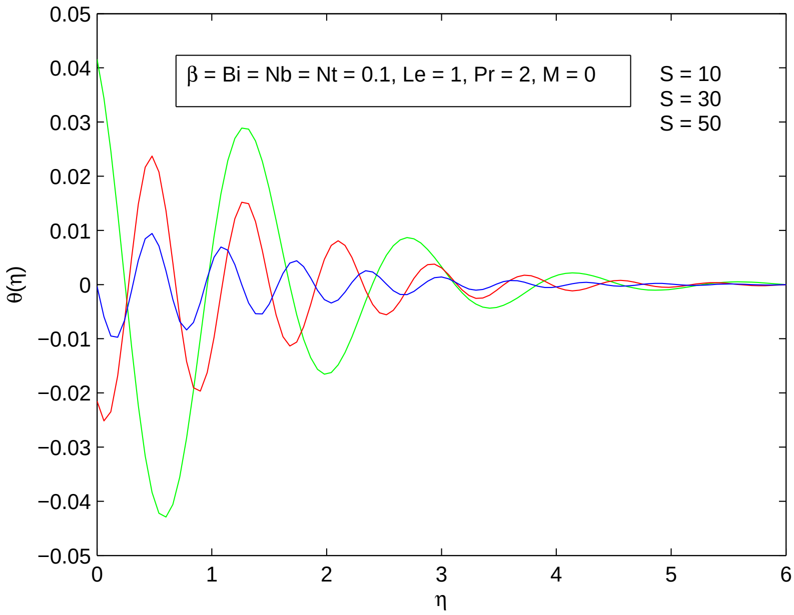

FIGURE 3.15: Impact of S temperature $\theta(\eta)$.

Chapter 4

Convection of steady slip Maxwell fluid and heat transfer with the effects of thermal radiation and chemical reaction over a stretching sheet

4.1 Introduction

In this chapter we extend the flow model of Cao et al. [24] that was introduced in previous chapter by considering the effect of thermal radiation as a function of temperature and chemical reaction as function of concentration. We will examine the steady slip flow with heat transfer of viscous, incompressible, laminar and two-dimensional fluid flow over a permeable plate through a porous surface with thermal radiation and chemical reaction. By using similarity transformation the nonlinear PDEs of momentum, temperature and concentration are converted into a system of ODEs. Numerical solution of these modeled ODEs are obtained by using shooting method. Finally at the end of chapter results are discussed and found to be in excellent agreement with Matlab bvp4c code. Significance of different physical parameters on dimensionless velocity, temperature and concentration are elaborated with the help of graphs and tables.

4.2 Problem formulation

Consider the laminar, two-dimensional and steady flow of a fluid with heat transfer past a stretching surface. The geometry of flow model is given in 4.1

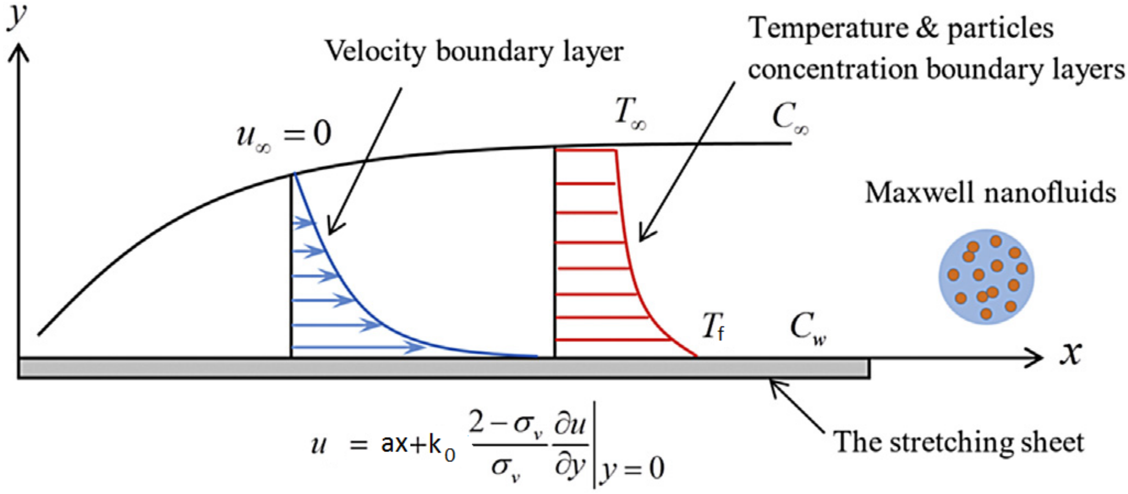

FIGURE 4.1: Geometry for the flow under consideration.

Assume that the fluid under investigation is taken as viscous and incompressible, the body forces like gravitational force, electromagnetic force, etc. are negligible. Thermal radiation are taken as function of temperature and chemical reaction as a function of concentration. The associated equations for the flow model are given in Eqs. (4.1) - (4.4), which under boundary layer approximation can be written as,

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{4.1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + k_0(\bar{u}^2 \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \bar{v}^2 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + 2\bar{u}\bar{v}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}), \tag{4.2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha \frac{\partial^2 T}{\partial \bar{y}^2} + \tau(D_B(\frac{\partial C}{\partial \bar{y}}\frac{\partial T}{\partial \bar{y}}) + \frac{D_T}{T_\infty}(\frac{\partial T}{\partial \bar{y}})^2) - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial \bar{y}}, \tag{4.3}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = D_B(\frac{\partial^2 C}{\partial \bar{y}^2}) + \frac{D_T}{T_\infty}(\frac{\partial^2 T}{\partial \bar{y}^2}) - k_1(C - C_\infty). \tag{4.4}$$

From Eqs. (4.1) - (4.4), $\bar{u}$ and $\bar{v}$ are the velocities in the $\bar{x}-$ and $\bar{y}-$ directions respectively, α denotes thermal diffusivity, base fluid density is ρ, ν denotes fluid kinematic viscosity, T denotes the fluid temperature, ambient temperature is T_∞, k_0 is the relaxation, Brownian diffusion coefficient D_B, D_T is the thermophoretic diffusion coefficient, c_p is the specific heat of the constant pressure, τ denotes the ratio of the effective heat capacity of the nanoparticle, q_r is radiative heat flux and nanoparticle volume fraction is C.

The associated boundary conditions for the above system of equations are,

$$\bar{u} = a\bar{x} + k_0\left(\frac{2-\sigma_\nu}{\sigma_\nu}\right)\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right), \ \bar{v} = 0, \ \ -k\frac{\partial T}{\partial \bar{y}} = h_f(T_f - T),$$

$$-D_B\frac{\partial C}{\partial \bar{y}} = h_c(C_w - C), \ \text{ at } \ \ y = 0, \tag{4.5}$$

$$\bar{u} \to 0, \quad T \to T_\infty, \quad C \to C_\infty, \text{as } \ \mathbf{y} \to \infty. \tag{4.6}$$

Here a is a constant, σ_ν is electrical conductivity, C_w is the fraction of nanoparticles at the wall, k is the thermal conductivity of nanofluid and C_∞ is the ambient volume fraction of nanoparticles.

The radiative heat flux q_r is given as

$$q_r = -\frac{4\sigma}{3k}\frac{\partial T^4}{\partial y} \tag{4.7}$$

where σ is Stefan Boltzmann constant, k is mean absorption coefficient. We expend T^4 by Taylor's series. $T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + ...,$
ignoring higher order terms, we get

$$T^4 = -3T_\infty^3 + 4T_\infty^3 T \tag{4.8}$$

substituting (4.8) into (4.7),we get

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty^4\sigma}{3k}\frac{\partial^2 T}{\partial y^2} \tag{4.9}$$

4.3 Solution of problem

In this section we transform the system of Eqs. (4.1) - (4.4) along with the boundary conditions (4.5) and (4.6) into a dimensionless form. To find out the solution of PDEs we use the similarity transformation technique here. We introduce the dimensionless similarity variable,

$$x = \frac{\bar{x}}{\sqrt{\frac{\nu}{a}}}, \quad y = \frac{\bar{y}}{\sqrt{\frac{\nu}{a}}}, \quad u = \frac{\bar{u}}{\sqrt{a\nu}}, \quad v = \frac{\bar{v}}{\sqrt{a\nu}}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}.$$

The stream function ψ defined by $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$. The equation of continuity (4.1) is satisfied identically, the effect of stream function on the remaining three equations, the momentum Eq. (4.2), the temperature Eq. (4.3) and concentration Eq. (4.4) are as follows,

$$\psi_x\psi_{xy} - \psi_x\psi_{yy} - \psi_{yyy} - \beta(\psi_y)^2\psi_{xxy} + (\psi_x)^2\psi_{yyy} - 2\psi_x\psi_y\psi_{xyy} = 0, \tag{4.10}$$

$$(1 + Tr)\theta_{yy} + Pr(Nt(\theta_y)^2 + Nb\psi_y\theta_y + \psi_x\theta_y), \tag{4.11}$$

$$\phi_{yy} + \frac{Nt}{Nb}\theta_{yy} + LePr\psi_x\phi_y - LePr\gamma\psi. \tag{4.12}$$

where $\beta = ak_0$ is the Maxwell parameter, $Tr = \frac{16\sigma T_\infty^4}{3kk_\infty}$ is thermal radiation parameter, $Nb = \frac{\tau D_B(C_w - C_\infty)}{\nu}$ is the Brownian motion parameter, $Nt = \frac{\tau D_T(T_f - T_\infty)}{\nu T_\infty}$ is the thermophoresis parameter, $Le = \frac{\alpha}{D_B}$ is Lewis number $Pr = \frac{\nu}{\alpha}$ denotes Prandtl number and $\gamma = \frac{k}{a}$ is chemical reaction parameter . After applying the stream function, the corresponding boundary conditions for the velocity components expressed in Eqs. (4.5) and (4.6) would be,

$$\psi = 0,\ \psi_y = 1 + b\psi_{yy},\ \theta_y = Bi1(\theta - 1),\ \phi_y = Bi2(\phi - 1),\ \text{at } y = 0, \tag{4.13}$$

$$\psi_y \to 0,\quad \theta \to 0,\quad \phi \to 0, \text{as}\quad y \to \infty. \tag{4.14}$$

where $b = k_0\frac{2-\sigma_\nu}{\sigma_\nu}\sqrt{\frac{a}{\nu}}$ is slip coefficient, $Bi1 = \frac{h_f}{k}\sqrt{\frac{\nu}{a}}$ is the temperature Biot number and $Bi2 = \frac{h_f}{D_B}\sqrt{\frac{\nu}{a}}$ is concentration Biot number.
The corresponding variable and functions are

$$\eta = y,\quad \psi = xG(\eta),\quad \theta = \theta(\eta),\quad \phi = \phi(\eta). \tag{4.15}$$

The differential Eqs. (4.10) - (4.12) with the associated boundary conditions (4.13) and (4.14) takes the following form after applying the similarity transformation together with stream function,

$$(1 + \beta G^2)G''' + GG'' - (G')^2 + 2\beta GG'G'' = 0, \tag{4.16}$$

$$(1 + Tr)\theta'' + PrNb\theta'\phi' + PrNt(\theta')^2 + PrG\theta' = 0, \tag{4.17}$$

$$\phi'' + \frac{Nt}{Nb}\theta'' + LePrG\phi' - LePr\gamma\phi = 0, \tag{4.18}$$

$$G(\eta) = 0,\ G'(\eta) = 1 + bG''(\eta),\ \theta'(\eta) = Bi1(\theta(\eta) - 1),$$

$$\phi'(\eta) = Bi2(\phi(\eta) - 1),\quad \text{at}\quad \eta = 0, \tag{4.19}$$

$$G'(\eta) \to 0,\quad \theta(\eta) \to 0,\quad \phi(\eta) \to 0,\quad \text{at}\quad \eta \to \infty. \tag{4.20}$$

4.4 Solution methodology

The analytic solution of system of equations with corresponding boundary conditions (4.16) - (4.18) cannot be found because they are non linear and coupled. So we use numerical technique, i.e., shooting - Newton technique with fourth order Runge-Kutta method [25]. In order to solve the system of ordinary differential Eqs. (4.16) - (4.18) with boundary conditions (4.19) - (4.20) using shooting method, we have to convert these equations into a system of first order differential equations, let

$$\begin{aligned} &G = y_1, \quad G' = y_2, \quad G'' = y_3, \quad G''' = y_3', \\ &\theta = y_4, \quad \theta' = y_5, \quad \theta'' = y_5', \\ &\phi = y_6, \quad \phi' = y_7, \quad \phi'' = y_7'. \end{aligned} \tag{4.21}$$

Then the coupled nonlinear momentum, temperature and concentration equations are converted into system of seven first order simultaneous equations and the corresponding boundary conditions transforms the following form:

$$y_1' = y_2, \tag{4.22}$$

$$y_2' = y_3, \tag{4.23}$$

$$y_3' = \frac{1}{1+\beta y_1^2}(-y_1 y_3 - 2\beta y_1 y_2 y_3 + (y_2)^2), \tag{4.24}$$

$$y_4' = y_5, \tag{4.25}$$

$$y_5' = \frac{1}{1+Tr}(-PrNby_5 y_7 - PrNt(y_5)^2 - Pry_1 y_5), \tag{4.26}$$

$$y_6' = y_7, \tag{4.27}$$

$$y_7' = -LePry_1 y_7 + LePr\gamma y_6 - \frac{Nt}{Nb} y_5', \tag{4.28}$$

$$y_1(0) = 0,\ y_2(0) = 1 + by_3(0),\ y_5(0) = Bi1(y_4(0) - 1),\ y_7(0) = Bi2(y_6(0) - 1). \tag{4.29}$$

The shooting method requires the initial guess for $y_3(\eta)$, $y_4(\eta)$ and $y_6(\eta)$ at $\eta = 0$, and through Newton's method we vary each guess until we obtain an appropriate solution for our problem. To check accuracy we compare obtained result by the numerical results acquired by Matlab bvp4c solver and found them in excellent agreement.

4.5 Results and discussion

The objective of this section is to analyze the effect of different parameters, b, β, Pr, Tr, Nb, Nt, Le, γ, $Bi1$ and $Bi2$ (i.e., slip coefficient, Maxwell parameter, Prandtl number temperature, thermal radiation parameter, Brownaian motion parameter, Thermophoresis parameter, Lewis number, Biot number and thermal relaxation time .) on dimensionless velocity, dimensionless temperature and dimensionless concentration profiles.

In this suction, we discuss some parameters in the form of tables. In Table 4.1 we discuss about the effect of Maxwell parameter β on $-G''(0)$, $-\theta'(0)$ and $-\phi'(0)$. The increase of Maxwell parameter may cause increase in $-G''(0)$ and increase of Maxwell parameter may cause decrease in $-\theta'(0)$ and $-\phi'(0)$. Table 4.2 shows the effect of Prandtl number, Biot number and Lewis number on $-\theta'(0)$ and compare the result with the obtained results of Makinde and Aziz [6]. We observe that increase of Biot number may cause increase in $-\theta'(0)$ and converse for Lewis number.

	Shooting			**Bvp4c**		
$\boldsymbol{\beta}$	$-G''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-G''(0)$	$-\theta'(0)$	$-\phi'(0)$
0	0.86262	0.08531	0.00760	0.86262	0.08536	0.00763
0.2	0.89747	0.08508	0.00758	0.89747	0.08508	0.00758
0.5	0.94436	0.08507	0.00711	0.94436	0.08507	0.00711
0.8	0.98676	0.08440	0.00716	0.98682	0.08440	0.00716
1	1.01413	0.08430	0.00691	1.01423	0.08430	0.00691

TABLE 4.1: Values of $-G''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for $Nb = Nt = b = Bi = 0.1, Pr = Le = 1$

Parameters			**Present**	**Makinde and Aziz**[6]
$\boldsymbol{Pr}$	$\boldsymbol{Bi}$	$\boldsymbol{Le}$	$-\theta'(0)$	$-\theta'(0)$
1	0.1	5	0.07895	0.0789
2			0.08063	0.0806
3			0.07358	0.0735
4			0.03872	0.0387
5	1		0.1476	0.1476
	10		0.15508	0.1550
	100		0.15572	0.1557
	∞		0.15577	0.1557
	0.1	10	0.06473	0.0647
		15	0.06008	0.0600
		20	0.05704	0.0570

TABLE 4.2: Comparison of $-\theta'(0)$ for $Nb = Nt = 0.5$

In the following section, we discuss the impact of different parameters on velocity, temperature and concentration profile. We start with the effect of Maxwell parameter β on velocity $G'(\eta)$ shown in Figure 4.2. $G'(\eta)$ increases with the increase of Maxwell parameter β. So $G'(\eta)$ is increasing function of β. Figure 4.3 shows the effect of slip coefficient b on $G'(\eta)$. In Figure, velocity $G'(\eta)$ shows decreasing behaviour for the increment of slip coefficient b. The effect of thermal radiation parameter Tr on temperature profile $\theta(\eta)$ are shown in Figure 4.4 respectively. Temperature increases with the increase of thermal radiation parameter Tr. The effect of radiation is intensify the heat transfer thus radiation should be at its minimum in order to fascilate cooling process. In Figure 4.5 shows the effect of Prandtl number on temperature. $\theta(\eta)$ decrease with the increase of Pr because Prandtl number is the ratio of momentum diffusivity to thermal diffusivity so when we increase Prandtl number thermal diffusivity decrease that cause decrease in temperature and thermal boundary layer thickness. Figure 4.6 shows the effect of Prandtl number Pr on concentration $\phi(\eta)$. $\phi(\eta)$ decreases with the increase of Pr. The decrease of $\phi(\eta)$ due to increase of Pr is far away from surface. Thermophoresis parameter Nt is increasing parameter of both temperature and concentration are shown in Figure 4.7 and 4.8. This is because thermophoretic force generated by the temperature gradient results in fast flow away from the stretching surface. In Figure 4.10, Brownaian motion parameter Nb is decreasing function of $\phi(\eta)$. Concentration decreases with the increase of Nb near the surface. Temperature Biot Number $Bi1$ is increasing function of both temperature and concentration as shown in Figure 4.11 and 4.12. In Figure 4.13, concentration decreases with the increase of concentration Biot number $Bi2$. Lewis number Le is decreasing function of $\phi(\eta)$. Increase Lewis number may decrease Brownian diffusion coefficient because Lewis number is the ratio of momentum diffusivity to Brownian diffusion coefficient. Thermal relaxation parameter γ is decreasing function of concentration shown in Figure 4.14 and 4.15 respectively.

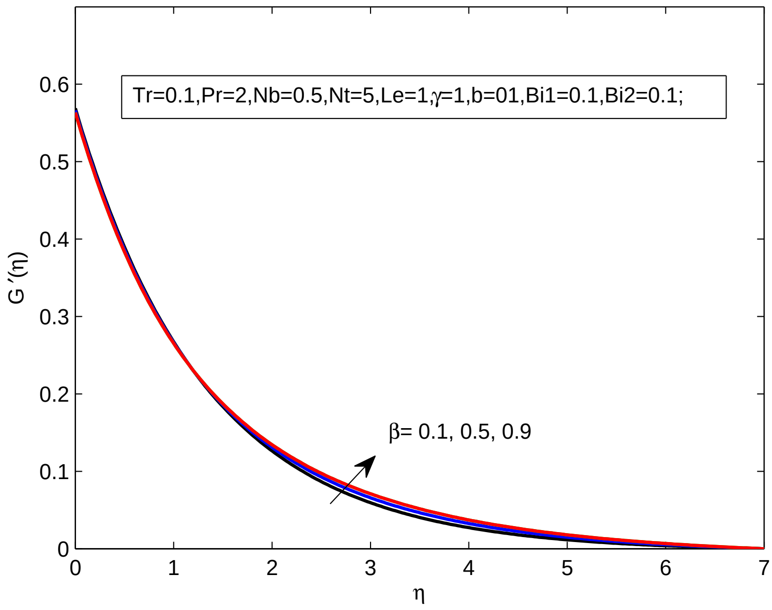

FIGURE 4.2: impact β on velocity $G'(\eta)$

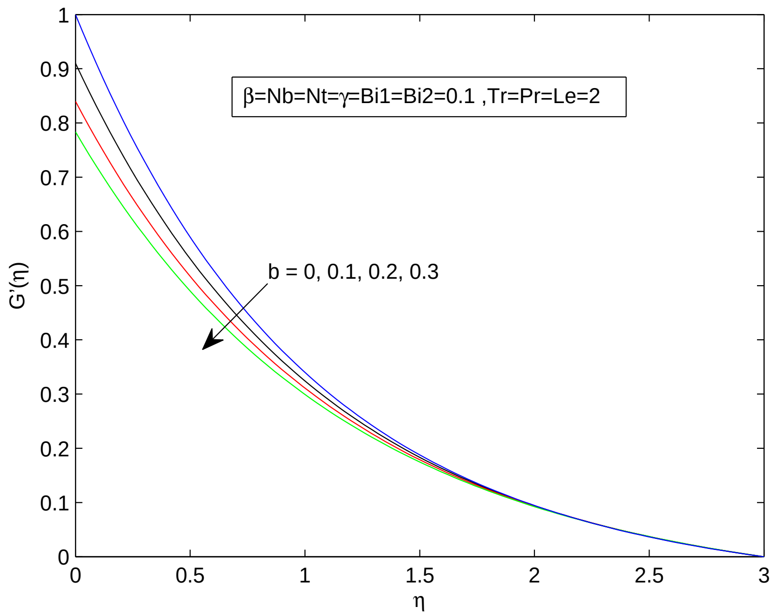

FIGURE 4.3: impact of slip coefficient b on velocity $G'(\eta)$

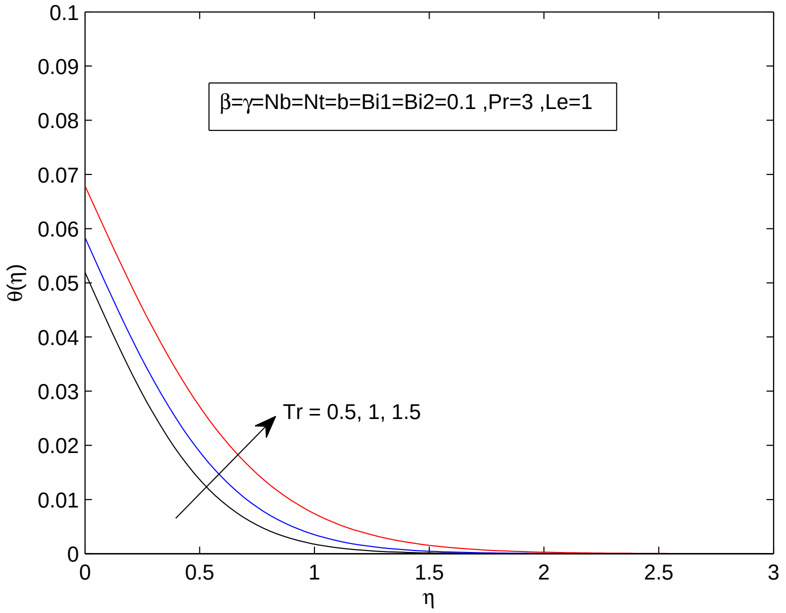

FIGURE 4.4: Impact of Tr on temperature $\theta(\eta)$

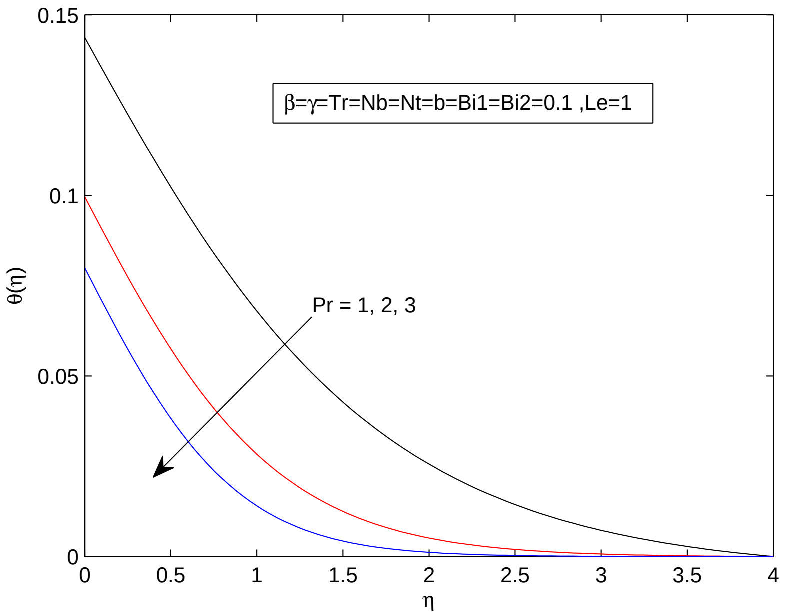

FIGURE 4.5: Impact of Pr on temperature $\theta(\eta)$

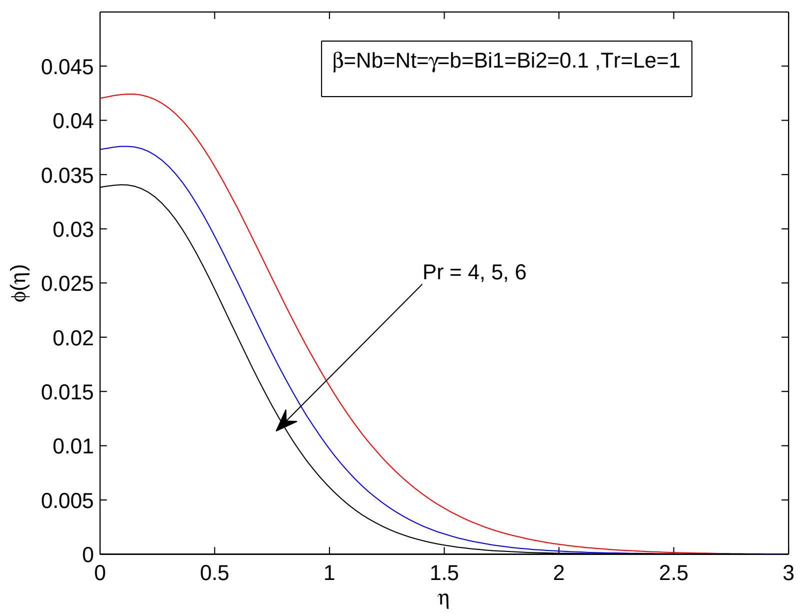

FIGURE 4.6: Impact of Pr on $\phi(\eta)$

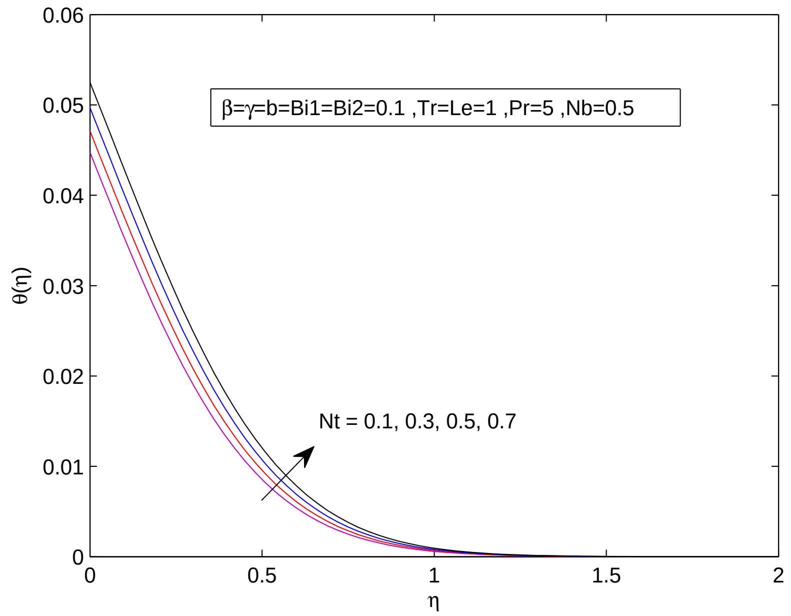

FIGURE 4.7: Impact of Nt on $\theta(\eta)$

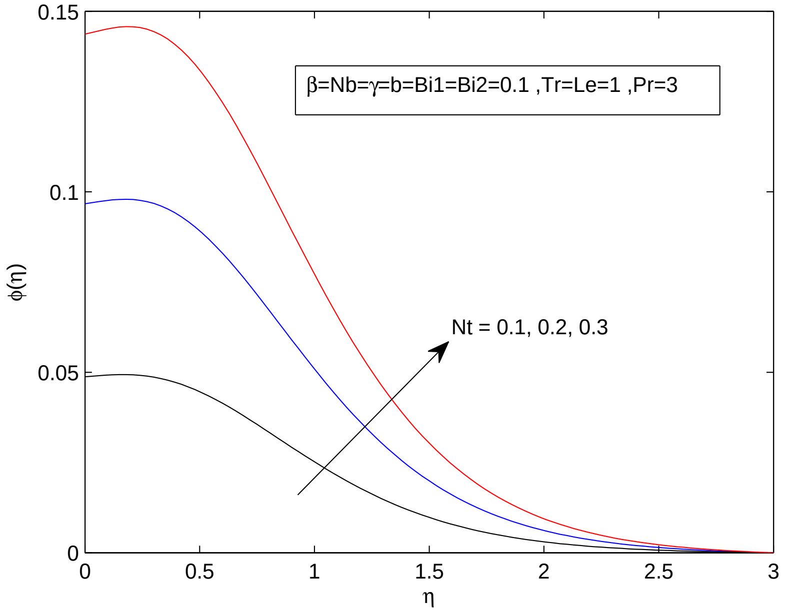

FIGURE 4.8: Impact of Nt on $\phi(\eta)$

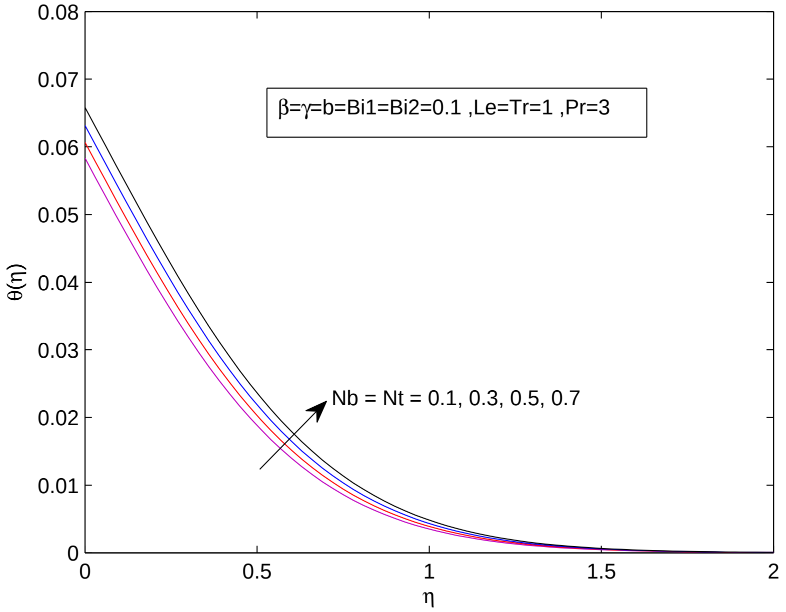

FIGURE 4.9: Impact of Nb and Nt on $\theta(\eta)$

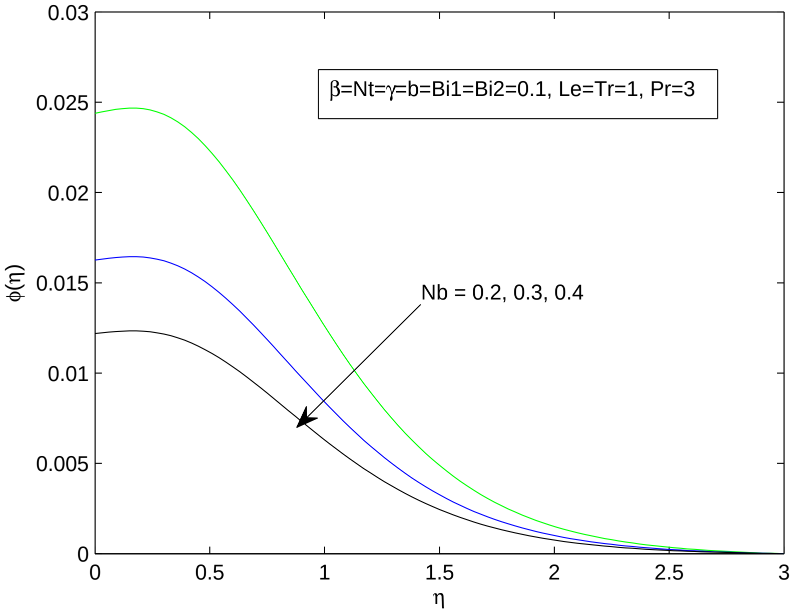

FIGURE 4.10: Impact of Nb on $\phi(\eta)$

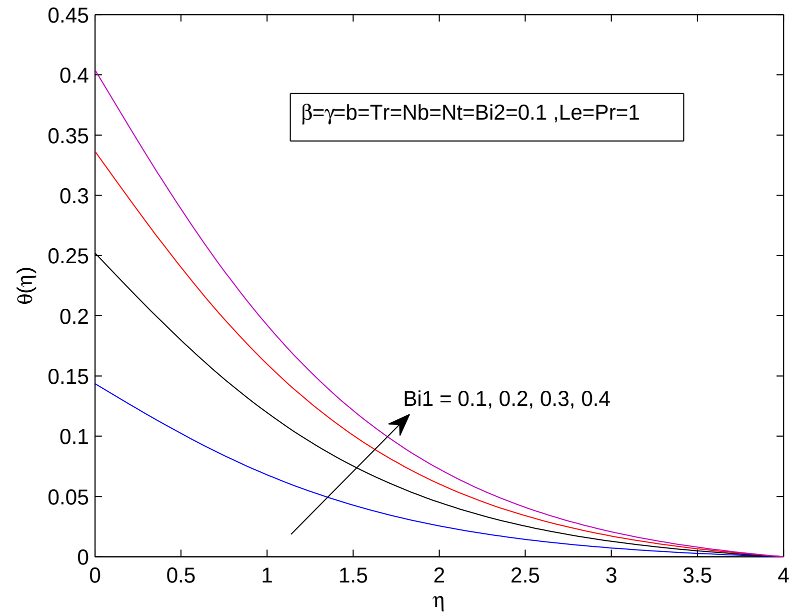

FIGURE 4.11: Impact of $Bi1$ on $\theta(\eta)$

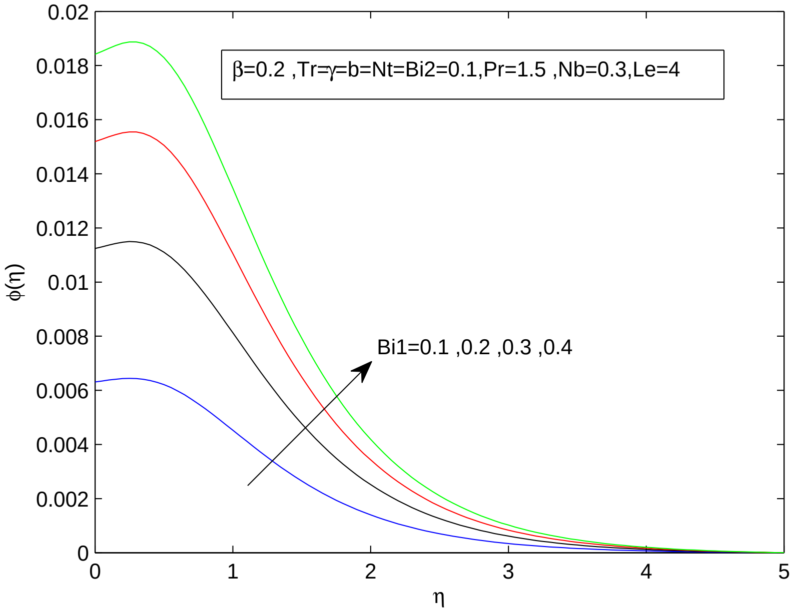

FIGURE 4.12: Impact of $Bi1$ on $\phi(\eta)$

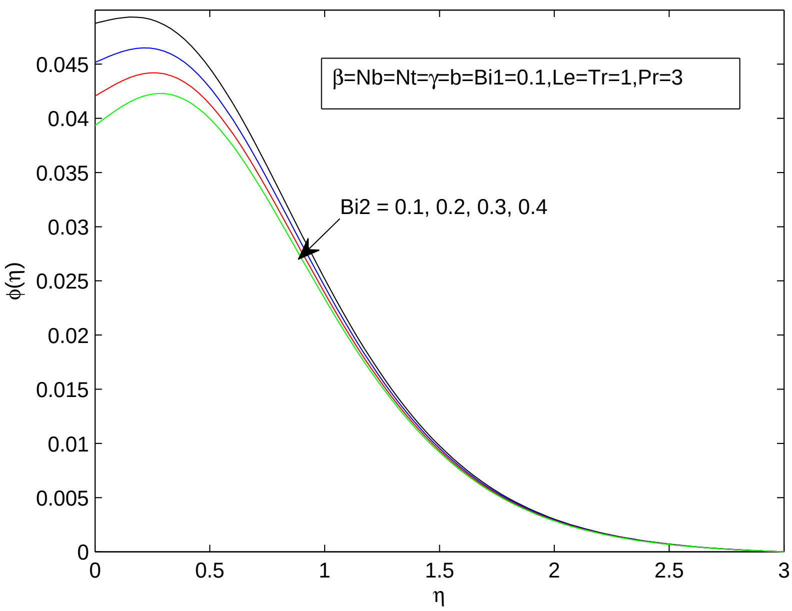

FIGURE 4.13: Impact of $Bi2$ on $\phi(\eta)$

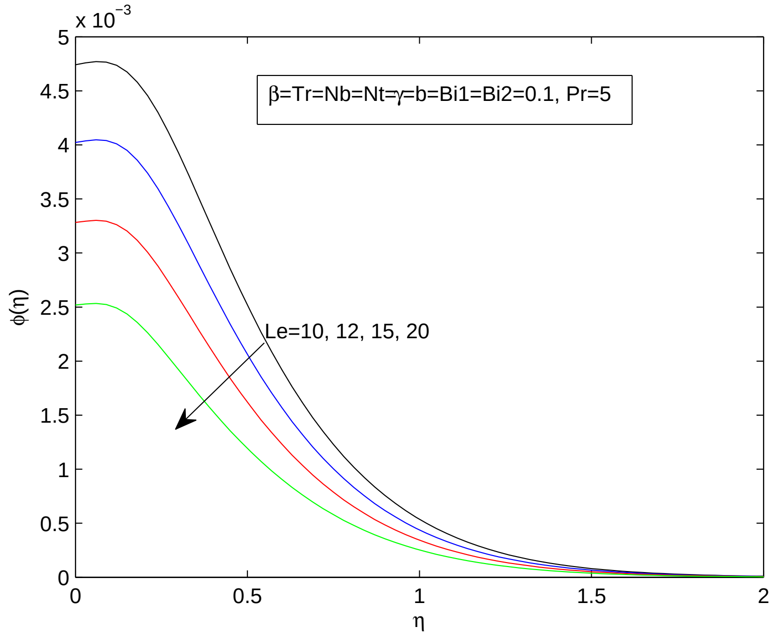

FIGURE 4.14: Impact of Le on $\phi(\eta)$

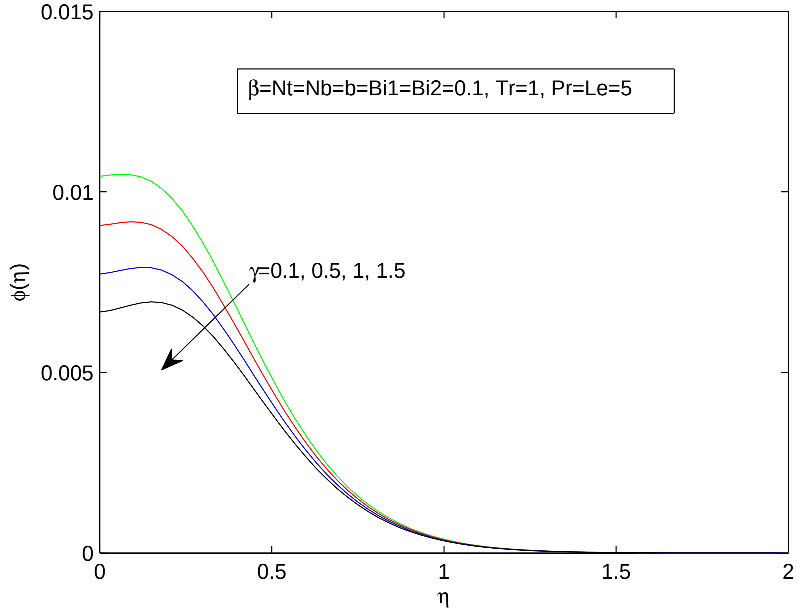

FIGURE 4.15: Impact of γ on $\phi(\eta)$

Chapter 5

Conclusion

We have presented the numerical study of steady slip flow and heat transfer of viscous, incompressible, laminar and two-dimensional fluid over a stretching porous surface with thermal radiation and chemical reaction. The current investigation is carried out in the presence of velocity, thermal slip conditions and convective boundary condition. The effects of different physical parameters on dimensionless velocity G' and dimensionless temperature θ and concentration ϕ are presented in the form of tables and graphs. The main findings of this work are as follows

- Velocity field G' increases by enlarging Maxwell parameter β and decreases by enlarging slip coefficient b.
- Temperature field θ increases with an increase in thermal radiation Tr.
- Increase of Prandtl number Pr causes decrease in temperature and concentration.
- Temperature and concentration increase by enlarging thermophoresis parameter Nt
- For larger values of Lewis number Le and chemical reaction parameter γ, concentration field ϕ shows decreasing behavior.

5.1 Future recommendations.

The present model has shown many simplifications to focus on the principal effects of slip parameter, thermal radiation and chemical reaction. An interesting area to investigate in future can be the use of thermal radiation, second order slip at the boundary, impact of different nanoparticles, viscous dissipation and variable porosity.

Bibliography

[1] SUS Choi. Enhancing thermal conductivity of fluids with nanoparticles. *ASME-Publications-Fed*, 231:99–106, 1995.

[2] J. A. Eastman, S. R. Phillpot, S. U. S. Choi, and P. Keblinski. Thermal transport in nanofluids. *Annu. Rev. Mater. Res.*, 34:219–246, 2004.

[3] J. Buongiorno. Convective transport in nanofluids. *J HEAT TRANSF*, 128(3): 240–250, 2006.

[4] A. V. Kuznetsov and D. A. Nield. Natural convective boundary-layer flow of a nanofluid past a vertical plate. *INT J THERM SCI*, 49(2):243–247, 2010.

[5] A. V. Kuznetsov and D. A. Nield. Natural convective boundary-layer flow of a nanofluid past a vertical plate: A revised model. *INT J THERM SCI*, 77:126–129, 2014.

[6] O. D. Makinde and A. Aziz. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. *INT J THERM SCI*, 50(7):1326–1332, 2011.

[7] G. K. Ramesh and B. J. Gireesha. Influence of heat source/sink on a maxwell fluid over a stretching surface with convective boundary condition in the presence of nanoparticles. *Ain Shams Eng. J*, 5(3):991–998, 2014.

[8] N. F. M. Noor. Analysis for mhd flow of a maxwell fluid past a vertical stretching sheet in the presence of thermophoresis and chemical reaction. *World Acad. Sci., Eng. Technol*, 64:1019–1023, 2012.

[9] V. A. Akbar, A. Alizadeh-Pahlavan, and K. Sadeghy. The influence of thermal radiation on mhd flow of maxwellian fluids above stretching sheets. *Communications in Nonlinear Science and Numerical Simulation*, 14(3):779–794, 2009.

[10] T. Hayat, C. Fetecau, and M. Sajid. On MHD transient flow of a maxwell fluid in a porous medium and rotating frame. *Physics letters A*, 372(10):1639–1644, 2008.

[11] M. Kumari and G. Nath. Steady mixed convection stagnation-point flow of upper convected maxwell fluids with magnetic field. *INT J NONLIN MECH*, 44(10): 1048–1055, 2009.

[12] K. Bhattacharyya, S. Mukhopadhyay, and G. C. Layek. Steady boundary layer slip flow and heat transfer over a flat porous plate embedded in a porous media. *J PETROL SCI ENG*, 78(2):304–309, 2011.

[13] K. Das. Slip effects on heat and mass transfer in MHD micropolar fluid flow over an inclined plate with thermal radiation and chemical reaction. *INT J NUMER METHOD F*, 70(1):96–113, 2012.

[14] K. Das. Slip flow and convective heat transfer of nanofluids over a permeable stretching surface. *Computers & Fluids*, 64:34–42, 2012.

[15] I. C. Liu. Flow and heat transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet subject to a transverse magnetic field. *INT J NONLIN MECH*, 40(4):465–474, 2005.

[16] I. C. Liu. A note on heat and mass transfer for a hydromagnetic flow over a stretching sheet. *INT COMMUN HEAT MASS*, 32(8):1075–1084, 2005.

[17] P. G. Siddheshwar and U. S. Mahabaleswar. Effects of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet. *INT J NONLIN MECH*, 40(6):807–820, 2005.

[18] T. Hayat, Z. Abbas, M. Sajid, and S. Asghar. The influence of thermal radiation on mhd flow of a second grade fluid. *INT J HEAT MASS TRAN*, 50(5):931–941, 2007.

[19] K. Bhattacharyya, S. Mukhopadhyay, and G. C. Layek. Steady boundary layer slip flow and heat transfer over a flat porous plate embedded in a porous media. *J PETROL SCI ENG*, 78(2):304–309, 2011.

[20] K. Das. Slip effects on heat and mass transfer in MHD micropolar fluid flow over an inclined plate with thermal radiation and chemical reaction. *INT J NUMER METHOD F*, 70(1):96–113, 2012.

[21] A. Noghrehabadi, R. Pourrajab, and M. Ghalambaz. Effect of partial slip boundary condition on the flow and heat transfer of nanofluids past stretching sheet prescribed constant wall temperature. *INT J THERM SCI*, 54:253–261, 2012.

[22] L. Zheng, J. Niu, X. Zhang, and Y. Gao. MHD flow and heat transfer over a porous shrinking surface with velocity slip and temperature jump. *MATH COMPUT MODEL*, 56(5):133–144, 2012.

[23] R. Sharma, A. Ishak, and I. Pop. Partial slip flow and heat transfer over a stretching sheet in a nanofluid. *MATH PROBL ENG*, 2013, 2013.

[24] L. Cao, X. Si, and L. Zheng. Convection of maxwell fluid over stretching porous surface with heat source/sink in presence of nanoparticles: Lie group analysis. *APPL MATH MECH*, 37(4):433–442, 2016.

[25] T. Y. Na. Computational methods in engineering boundary value problems (1979). *Academic, New York*, 1979.